Effects from Pandemic and Stagnation

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12 March, 2021

Abstract

We discuss the impact of shocks from a sudden pandemic in a recessionary economy. The framework is one of imperfect knowledge with a standard New Keynesian model. Global aspects of dynamics with possible interest-rate lower bound are discussed.

JEL classification: E62, E63, E52, D84, E71.

Key words: Stagnation, Expectations, Productivity shocks, Adaptive Learning, New-Keynesian Model.

1 Introduction

The global financial crisis of 2008-09 dealt a major blow to most market economies. It displaced many economies from their trend growth path and even at present many countries have not been able to get back to their pre-2008 trend levels. This period of stagnation can be viewed as an episode of very slow recovery or ongoing stagnation, see the comments on the literature below. The covid pandemic that struck in 2020 can be seen as generating further macroeconomic shocks that led to worsening macroeconomic developments for countries still recovering from the 2008-09 crisis.

We begin by looking at basic macro data from 2005 to present. This period covers the aftermath of the 2008-9 financial crisis and also the very beginning of the crisis from the covid pandemic.

Figure 1 illustrates the post-2008 development of GDP per capita for the United States of America and the Euro area. It is seen that the 2008-9 financial crisis resulted in sharp recession in both economies. For the US, the decrease from 2007Q4 to 2009Q2 was about 6.0%. For the euro area the drop in GDP per capita from 2008Q1 to 2009Q2 was 5.5%. The US and Euro area economies started to recover quickly after the rapid decline of GDP in 2008, but neither has succeeded in returning to its pre-2008 trend. The data for the beginning of 2020 shows a new decline in GDP per capita due to the covid pandemic. The decline appears to be even bigger than 2008 recession. Unfortunately, it is not possible at present to have a complete picture as the pandemic is ongoing.

FIGURE 1 ABOUT HERE

Figure 2 shows the monthly development of consumer prices in the US and Euro area since 2005. The data describes core inflation, i.e. the relatively volatile food and energy prices are excluded from the indices. The data is seasonally and work-day adjusted. Looking at the period 2008 and after, it is seen that during the financial crisis US price index shows a couple of short intervals with basically constant prices, but otherwise the US index of core prices has risen at relatively constant rate. In the Euro area the core price index has continuously increased but its rate of change decreased in the 2009-10 time interval and more systematically since 2013. The beginning of 2020 marks possible additional variability for the US and Euro area core consumer prices. The data does not show actual deflation even if the risk of deflation has been discussed.

FIGURE 2 ABOUT HERE

The development of interest rates has been remarkable as a very rapid decline of policy interest rates to near zero level occurred in the 2008-9 crisis, see Figure 3 below. Both the US Federal Reserve (US Fed) and the European Central Bank (ECB) lowered their policy rate to approximately zero (and later even slightly below zero by Euro area). Policy rates have, on the whole, remained at remarkably low levels since the financial crisis. US Fed lowered its policy rate very quickly to 0.25 percent (on average). It then began to increase very gradually the rate in 2016 and by 2019 the rate reached 2.5 percent as an attempt to get to normal levels of the policy rate This process was abruptly reversed when an onset of the covid crisis became inevitable. The ECB lowered its policy rate very quickly to one percent, but its subsequent decisions became gradualist until 2015, when the ECB policy rate was reduced to zero and then in 2016 even to slightly negative level. The ECB did not have an opportunity to attempt a move to more normal policies before the start of the covid crisis.

The extensive period of very low levels of policy interest rates is unusual as it can be argued that it is not possible to lower these rates much below zero.[1] For convenience we refer to the approximately zero interest rates as the ZLB (zero lower bound).

FIGURE 3 ABOUT HERE

The remarkable macroeconomic phenomena since 2008-9 have led to different ways of thinking about the period since 2008-9 and alternative approaches have been introduced to study the post-crisis macroeconomic developments. One possibility is to see the 2008-9 crisis as a persistent major shock to economic fundamentals that have displaced the economies from their normal time path for a significant period of time. In this approach it is thought that the economy will return to normality once the shock has subsided. In the literature a persistent exogenous shock to the discount rate of households or, more plausibly, emergence of a credit-spread is seen as the fundamental reason for very low interest rates and sluggish GDP development. These shocks have been emphasized for example by Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), Corsetti, Kuester, Meier, and Muller (2010) and Woodford (2011). While this approach has been fruitful in suggesting suitable monetary and fiscal policy responses to such shocks, it has several somewhat unattractive features. It relies heavily on the persistence of a shock that evaporates according to an exogenous process, and recession ends as soon as the exogenous negative shock ends. Furthermore, this approach does not do justice to an independent role for expectations.

A different view is to think of the post-2008 period as a new macroeconomic regime, ongoing (or 'secular') stagnation.[2] The stagnation regime was effectively initiated by negative shocks that created uncertainties and pessimism to economic expectations. In standard New Keynesian macroeconomic models stagnation can be a possible macroeconomic regime that arises from a second self-fulfilling steady state due to the ZLB. This approach, developed by Benhabib, Schmitt-Grohe, and Uribe (2001) and first used by Reifschneider and Williams (2000), emphasizes the existence of multiple rational expectations equilibria (REE) when the interest-rate rule is subject to the ZLB. Bullard (2010) suggests the possibility of the second equilibrium using data for the US and Japan.

[1] A sufficiently negative rate would lead to hoarding of cash, which would have stability issues for the financial system.

[2] For different arguments and explanations about long-lasting stagnation see, for example, Summers (2013), Teulings and Baldwin (2014), Eggertsson, Mehrotra, and Robbins (2019) and Benigno and Fornaro (2018).

A major problem with this second approach is its neglect of the association of the ZLB with periods of recession, low output and stagnation. Although there is a long-run trade-off in the NK model between output and inflation, the extent of this trade-off is quite minor: at the unintended low inflation steady state the level of aggregate output is only very slightly below that of the intended steady state. However, real GDP per capita since 2001 for the US, Japan and the euro area, clearly illustrates the association of depressed output levels in these countries with the ZLB. This is inconsistent with the view of two steady-states in the second approach.

In this paper we employ the stagnation approach and use a nonlinear New Keynesian model with imperfect knowledge and learning to study consequences of the covid shocks to stagnation dynamics.[3] As already noted, the onset of the covid pandemic is seen as adding new negative shocks to the aftermath of shocks from the global financial crisis. We consider two types of negative shocks: (i) persistent shock of pessimism to expectations of aggregate output and (ii) persistent negative shock to total factor productivity. Dynamics of more severe stagnation arising from the covid shocks are then described and possibilities for macroeconomic policies to improve outcomes are considered. Our analysis is very much a first approach, as a one-sector model is used and dynamics of a health epidemic are not integrated to macroeconomic dynamics.

2 Imperfect Knowledge and Learning

This paper belongs to the line of research that emphasizes imperfect knowledge and learning behavior in the formation of expectations about the future. This is in contrast to the hypothesis of rational expectations (RE) which has been the main paradigm for modelling expectation formation in macroeconomics over the past several decades. RE is usually formulated as complete knowledge of the systematic aspects of the economy including the underlying model structure and parameters. This assumption is sometimes restrictive since, in reality, economic and policy decisions are made under incomplete knowledge about the underlying structure and parameters.

There is increasing interest in studying situations where agents have much more imprecise knowledge than is presumed by RE. One may assume that economic agents

[3] The RE viewpoint to multiple equilibria has some difficulties, which can be avoided by modelling expectations as arising from adaptive learning behavior, see the discussion in Evans, Honkapohja, and Mitra (2020). The approach was developed by Evans, Guse, and Honkapohja (2008), Benhabib, Evans, and Honkapohja (2014).

try to improve their knowledge of the economy by using methods of scientific inference as they learn over time. During the 1990s the assumption that economic agents engage in adaptive learning behavior was incorporated into macroeconomic theory (see e.g. Evans and Honkapohja (2001)). The literature on adaptive learning has provided support for rational expectations by showing how, in many cases, boundedly rational agents with limited knowledge can converge to a rational expectations equilibrium (REE) in the long run. More specifically, provided an appropriate stability condition is satisfied, REE is an asymptotic outcome of the use of recursive least squares and related learning rules by private agents.

Furthermore, the typical situation analyzed in economic models is one in which no future change in structure (surprise or anticipated) is contemplated by economic actors (and/or policy makers). However, in practice, changes in economic structure (e.g. policy changes) do take place. Again the standard way to analyze such policy changes in economic models continues to be the assumption of RE. The benchmark assumption of RE is very strong and arguably unrealistic when analyzing the effect of structural or policy changes. Economic agents need to have complete knowledge of the underlying economic structure, both before and after the policy change. They must also fully and rationally incorporate this knowledge in their decision making, and do so under the assumption that other agents are equally knowledgeable and equally rational. The Covid pandemic may be thought of as a supply shock (say a negative shock to productivity as in Section 5.2 below) which changes the structure of the economy. Assuming RE after these sudden, large unpredictable changes to the economy is especially problematic. Adaptive learning instead becomes a much more plausible approach to analyze these situations.

The essence of the adaptive learning approach is that agents are assumed not to understand the general equilibrium considerations that govern the evolution of the central endogenous variables (e.g. labor and factor prices). Again the assumption of imperfect knowledge is particularly plausible in the face of sudden, large changes in the economy. When learning agents are assumed to forecast the key variables statistically using any number of a variety of reduced form estimation techniques. In the learning literature the benchmark is to assume recursive least-squares updating in which agents use a "perceived law of motion" (an econometric specification) that corresponds in functional form (but not parameter values) to the rational expectations equilibrium of interest. However, the learning approach is flexible and alternative assumptions include agents using underparametrized or pure time-series models.

In the context of infinite-horizon agents solving dynamic optimization problems, our approach can be viewed as a version of the "anticipated utility" approach formulated by Kreps (1998) and discussed in Sargent (1999) and Cogley and Sargent (2008).

At each time t, agents solve their dynamic optimization problem, given forecasts of the future based on an estimated forecasting model, and use the solution to make their time t decisions. At time t +1, agents update their forecasting model and their forecasts of future variables, and resolve their dynamic optimization problem in order to make their time t +1 decisions. In the anticipated utility approach recommended by Kreps, agents update their forecasts over time but do not take into account the fact that their forecasting model will be revised in future periods. Clearly, this is a bounded rationality approach, since a full Bayesian approach would recognize the uncertainty in the parameters of the estimated forecasting model. However, as noted by Cogley and Sargent (2008), a full Bayesian approach in macroeconomic settings is typically "too complicated to be implemented," and thus the anticipated utility approach is an appealing implementation of bounded rationality and often provides an excellent approximation of Bayesian decisions.

3 The Model

3.1 Household-producers

The model is a standard New Keynesian macroeconomic model in which nominal price stickiness arises from adjustment costs in price setting as suggested by Rotemberg (1982). We use the Rotemberg formulation rather than the Calvo (1983) model of price stickiness because it enables us to study global dynamics in the nonlinear system. The analysis is based on the model in Evans, Honkapohja, and Mitra (2020) which should be consulted for the formal details. Here we only outline the structure of the model.

There is a continuum of household-producers, who produce differentiated goods and are maximize present value of period utilities over an infinite horizon. Utility in each period depends (i) positively on the aggregate of private consumption and a weighted value of government consumption per capita, (ii) positively on beginning-of-period real balances, (iii) negatively on labor supply and (iv) negatively on price adjustment costs. Utility functions are identical across agents. The flow budget constraint (in real terms) states that resources are spent on consumption, end-of-period-real balances, end-of-period bond holding and payment of lump-sum tax. The recourses come from initial money balances, interest and principal from bond holdings and revenue from production activity. Households treat government spending per capita as exogenous. The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition.

Household expectations over the entire future are in general subjective and may not be rational in accordance with anticipated utility maximization. Formally, subjective expectations are expectations of nonlinear functions of future random variables with an unknown distribution. A specific form of bounded rationality is assumed, so agents use *point expectations*, i.e. agents treat the expectation of a nonlinear function of random variables as equal to the value of the nonlinear function at the point expectations. The quality of this approximation depends, of course, on the severity of nonlinearities and the size of the shock variances.

Production function $y_{t,i} = A_t h_{t,i}^{\alpha}$, for each good variety i depends on labor input $h_{t,i}$ by the household-producer and a random aggregate productivity variable A_t. Production functions including the realization of the value of A_t are taken to be identical across agents. Output is differentiated and firms operate under monopolistic competition. Household-firms face a downward-sloping demand curve with random constant elasticity of substitution ν_t between any two goods.

3.2 Government

For simplicity, the government is assumed to follow a balanced budget policy, so that lump-sum taxes are used to pay for government spending.[4] The discount factor is given by the product of one-period real (gross) interest rates which in turn is the ratio of nominal (gross) interest rate divided by (gross) inflation.

Monetary policy is specified as a forward-looking interest rate rule taking an exponential form

$$\begin{aligned} R_t &= R(\pi^e_{t+1}, y^e_{t+1}) && (1) \\ &= 1 + (R^* - 1)\left(\frac{\pi^e_{t+1}}{\pi^*}\right)^{BR^*/(R^*-1)} \left(\frac{y^e_{t+1}}{y^*}\right)^{\phi_y} && (2) \end{aligned}$$

where R_t denotes the gross nominal interest rate in period t and π^e_{t+1}, y^e_{t+1} denote inflation and output expectations in period $t+1$.[5] $B > 1$ and $\phi_y \geq 0$ are policy parameters and $R^* = \beta^{-1}\pi^*$ is the policy interest rate at the target steady state π^*. y^* is the level of output associated with π^*. As $R^* \geq 1$ this interest-rate rule satisfies the zero lower bound for net interest rates.

Market clearing in aggregate and for each goods variety hold in the usual way. It is assumed the government can require that its demand is always met, so we have $y_{t,i} \geq g_t(i)$ for all i and $y_t \geq g_t$. Government purchases are distributed equally to

[4] Similar analysis can be done with more general formulation of government budget constraint and policy. For example, see Benhabib, Evans, and Honkapohja (2014).

[5] Forecast of a variable is denoted using the superscript e.

the households. As government guarantees a subsistence level of consumption to households, agents are required to pay their taxes and hence must work to produce at least the amounts that the government purchases.

3.2.1 Pricing decisions

The price adjustment cost function $\Phi(\frac{P_{t,j}}{P_{t-1,j}})$ is assumed to be asymmetric in the inflation factor $P_{t,j}/P_{t-1,j}$ for variety j. Here $P_{t,j}$ is the price of goods variety j in period t. Decisions of consumption, production and pricing are identical for all agents and goods varieties in the representative agent economy with identical agents and expectations. The optimal pricing i.e. inflation is given by

$$\begin{aligned}\Phi'(\pi_t)\pi_t &= \zeta_t + \sum_{s=1}^{\infty} \beta^s \zeta^e_{t+s}, \text{ where} \qquad (3)\\ \zeta^e_{t+s} &= \frac{\nu^e_{t+s}}{\alpha}\left(\frac{y^e_{t+s}}{A^e_{t+s}}\right)^{(1+\varepsilon)/\alpha} - \left(\nu^e_{t+s}-1\right) y^e_{t+s} \times (y^e_{t+s} - (1-\xi) g^e_{t+s})^{-1},\end{aligned}$$

provided a transversality condition holds. β is the subjective discount factor and $0 < \xi < 1$ is a parameter in the agents' utility function. We will treat (3) as the pricing decision rule.

In (3) the forecast $\zeta^e_{t+s,i}$ by agent i as producer generally requires forecasts of paths for the exogenous variables ν_{t+s}, A_{t+s}, the fiscal policy variable g_{t+s}, aggregate output y_{t+s}, relative price $\frac{P_{t+s,i}}{P_{t+s}}$, market demand $y_{t,i}$ for variety i and the term in $c_{t+s,i} + \xi g_{t+s}$ that arises from marginal utility. (3) is a conditional decision-rule as $\frac{P_{t+s,i}}{P_{t+s}}$ includes a future decision variable of the firm i. Agents are assumed to use this conditional decision rule supplemented by forecasts of the future relative prices $\frac{P_{t+s,i}}{P_{t+s}}$. As a simplification it is assumed that $P^e_{t+s,i} = P^e_{t+s}$ for all i.[6] Agents are assumed to use adaptive learning based on observed $P_{t,i}/P_t$ to forecast their expected future relative price. Thus agents' future pricing decisions $P_{t+s,i}$ will not in general be consistent with what would be their optimal choices under current expectations.[7]

Similarly, if heterogeneous agents were allowed for, each agent would need to forecast its own output (demand) $y_{t+s,i}$ as well as aggregate output y_{t+s}. In the representative agent case these are identical and agents have learned this relationship. The marginal utility term $(y^e_{t+s} - (1-\xi)g^e_{t+s})^{-1}$ in (3) can then be conveniently

[6] P^e_{t+s} is forecasted aggregate price level for period $t+s$.

[7] Note that the agent's forecasts of future aggregate variables *will* in general be revised over time as new data become available.

forecasted using the market clearing condition. For fiscal policy we focus on the case in which the path of future government spending is credibly announced and is therefore known.

3.2.2 Consumption and temporary equilibrium

For the representative agent model the consumption function can be shown to be

$$\begin{aligned} c_t &= \max\left\{0, (1-\beta)\left[y_t - g_t\left(1+\frac{\xi\beta}{1-\beta}\right)\right]+\right. \\ &\qquad \left.(1-\beta)\sum_{s=1}^{\infty}\left(D^e_{t,t+s}\right)^{-1}\left(y^e_{t+s}-g^e_{t+s}(1-\xi)\right)\right\}. \end{aligned} \tag{4}$$

Letting r^e_{t+j} denote the forecasted gross real interest rate, $D^e_{t,t+s} = \prod_{j=1}^{s} r^e_{t+j}$ is the point expectation of the real discount factor for s periods into the future. For simplicity, agents are assumed to know the interest rate rule. y^e_{t+s} and g^e_{t+s} denote expected aggregate output (income) and government purchases per capita, respectively.

We collect the expectation variables, which are taken as given in the time t equilibrium: exogenous markup shocks $\{\nu^e_{t+s}\}$ and productivity shocks $\{A^e_{t+s}\}$, output $\{y^e_{t+s}\}$, government spending $\{g^e_{t+s}\}$, and inflation $\{\pi^e_{t+s}\}$, as well as the implied discount factors $D^e_{t,t+s}$. Temporary equilibrium consumption, output, interest rates and inflation are determined from (3), (4), interest rate rule and market clearing for period t.

Following the adaptive learning literature, our approach views aggregate dynamics as a sequence of temporary equilibria. At each point t, exogenous random variables are realized, economic agents form expectations of relevant future variables and their optimal decision rules are formed conditional on those expectations. Market clearing determines the time t temporary equilibrium levels of all variables including aggregate output and inflation. In the subsequent period $t+1$ new values of exogenous variables are realized, expectations are revised and a new temporary equilibrium is generated. Adaptive learning specifies how the forecast rules that are used to form expectations conditional on the information available are revised over time.

4 Dynamics of the Economy

We now consider how expectations are revised over time and the aggregate dynamics under adaptive learning. The key bounded-rationality assumption is that forecasts of future variables are made using the adaptive learning approach. Agents in the model are assumed to forecast like econometricians, regressing variables to be forecasted on observed explanatory variables and updating the forecast rule coefficients as new data become available. Updating of the coefficients is done using the recursive least-square learning to expectation formation as developed in Bray and Savin (1986), Marcet and Sargent (1989) and Evans and Honkapohja (2001).

Because the model is nonlinear and stochastic, it is illuminating to begin with the non-stochastic case in which adaptive learning rules are particularly simple. The nonstochastic version of the model provides initial formal results and also intuition to the global picture of the dynamics of the economy. If the random shocks are small, the nonstochastic version gives an approximation for the mean dynamics of the model. The shocks are fixed to be constants $\nu_t = \nu > 1$ and $A_t = A > 0$ and also government spending and its forecasts are fixed and constant $g_{t+s} = g^e_{t+s} = \bar{g}$.

In the nonstochastic case agents' forecasting model reflects a steady state and agents' beliefs are thus about the long-run averages. Introducing the notation

$$y^e_{t+s} = y^e_t, \text{ and } \pi^e_{t+s} = \pi^e_t \text{ for all } s > 0$$

for expectations in period t over all future periods, adaptive learning rules for the non-stochastic case take the simple form

$$\begin{aligned} y^e_t &= y^e_{t-1} + \omega(y_{t-1} - y^e_{t-1}) \text{ and} && (5) \\ \pi^e_t &= \pi^e_{t-1} + \omega(\pi_{t-1} - \pi^e_{t-1}), && (6) \end{aligned}$$

where $0 < \omega < 1$ is the learning "gain" parameter. Adaptive learning usually focuses on cases with ω small and examines local stability of steady states for all sufficiently small $\omega > 0$. Adaptive-learning rules of the form (5)-(6) are often called "steady-state learning" since agents are estimating a mean rather than a more complex time series model. Different assumptions about the computation of the mean have been used in the literature. One possibility is the simple arithmetic mean (so that all data receive an equal weight, namely traditional "least-squares learning") while another is to allow for different weights on the data. It is assumed here that agents use exponential discounting of past data, an assumption commonly used in the learning literature when agents are concerned that structural changes may be occurring. Thus, e.g. agents use a weighted average of observed inflation rates to estimate their mean, which they use to forecast future inflation rates.

To elaborate further, the parameter ω in (5)-(6), measures the extent to which past data is discounted. Under this algorithm the relative weight on data j periods earlier is $(1-\omega)^j$, i.e. past data is discounted at rate $1-\omega$. The optimal choice of ω is not straightforward and is most naturally addressed in a stochastic framework, since it involves a trade-off of "filtering" and "tracking." Lower values of ω more effectively filter out random noise, while higher values of ω are better at tracking structural change. Because the optimal choice of ω in general, and in the current context, is not straightforward, ω is treated as a given parameter.[8]

To continue, in a perfect-foresight steady state $y_t = y^e = y$ and $\pi_t = \pi_t^e = \pi$, thus we have

$$(1-\beta)\Phi'(\pi)\pi = \frac{\nu}{\alpha}(y/A)^{(1+\varepsilon)/\alpha} - (\nu-1)\, y \times (y-(1-\xi)\bar{g})^{-1}. \tag{7}$$

The Fisher equation with the interest rate rule $R(\pi, y)/\pi = \beta^{-1}$ is the remaining steady state equation. As is well known, there is a targeted steady state with $\pi = \pi^*$ and the level of output y^* determined from (7) with $\pi = \pi^*$. It is also well known that, due to ZLB, requiring $y > \bar{g}$ and $R(\pi, y)/\pi = \beta^{-1}$ with $R'(\pi, y) < \beta^{-1}$ results in a second steady state (π_L, y_L) with π_L and y_L determined from equations $R(\pi, y)/\pi = \beta^{-1}$ and (7). Under the adopted model calibration $1 > \pi_L > \beta$ with $\pi_L \approx \beta$.[9] Finally, if output is constrained to the lower bound $y = g = \bar{g}$, then there exists a third, stagnation steady state, with inflation π_S (actually deflation) at this steady state determined from (7) with $y_S = \bar{g}$.

We now turn to stability of the three steady states π^*, π_L and π_S under adaptive learning.[10] Stability and instability properties of the three steady states are:
(i) The targeted steady state at (π^*, y^*) is locally stable under steady state learning, provided the policy parameter ϕ_y is not too large.
(ii) The steady state (π_L, y_L) is not locally stable under steady state learning if ϕ_y is not too large.
(iii) The steady state (π_S, y_S) is locally stable under steady state learning, provided ϕ_y is not too large.[11]

[8] See Evans and Honkapohja (2001), Chapter 14, for a discussion of the choice of ω in stochastic models with structural change.

[9] If the ZLB were binding at $\pi = \pi_L$, so that $R = 1$, then $\pi_L = \beta$ and there would be deflation.

[10] Loosely speaking, a stable steady state may be viewed as an asymptotic outcome of the use of recursive least squares and related learning rules by private agents.

[11] Condition that ϕ_y not be too large is standard and known to be necessary, with forward-looking interest rate rules, in order to avoid indeterminacy of the targeted steady state.

FIGURE 4 ABOUT HERE

Because we have fully specified the temporary equilibrium of the nonlinear system, the analysis can be extended to look at the global system under learning. Figure 4 shows a schematic phase diagram for the system which illustrates the target steady state (π^*, y^*), the liquidity trap steady state (π_L, y_L) and the stagnation steady state $(\pi_S, \bar{g})$.[12] The two steady states (π^*, y^*) and (π_L, y_L) have been widely discussed in the literature.[13] As noted above, (π^*, y^*) is locally stable under the learning dynamics, while (π_L, y_L) is locally unstable. (π_S, g) is locally stable under learning. At $(\pi_S, \bar{g})$ output $y = \bar{g}$ is at the minimal level, with households receiving only $\bar{g}$ as subsistence consumption (private consumption is zero). This steady state also involves rapid deflation.

For the targeted steady state (π^*, y^*) it is possible to construct a set Ω, called the domain of attraction in the expectations space of (π_t^e, y_t^e). Ω consists of all points $(\pi_t^e, y_t^e) \in \Omega$ such that the economy with (π_t^e, y_t^e) as the initial conditions will converge under learning to the targeted steady state (π^*, y^*). Figure 4 illustrates Ω so that the curve SS is its outer boundary. Note that there is divergence to the stagnation steady state from all points outside Ω, except for the 1-dimensional curve SS which is the global stable manifold of (π_L, y_L).

Thus there is a real possibility that after significant shocks, in the form of an adverse shift in expectations (π^e, y^e), the economy moves into and becomes stuck in a region leading to stagnation under unchanged policy with government spending remaining constant and the central bank adhering to its interest-rate rule and ZLB.

Figure 4 illustrates some challenges in the design of fiscal and monetary policy.[14] It is evident that if the economy is within the stagnation region, then sufficiently aggressive policy needs to be taken so that dynamics are transferred to inside the domain. Clearly the size of the required policy change will depend on the initial position (π^e, y^e) following the shock and thus choosing the magnitude of the policy can be delicate.

It is important to note that the preceding analysis is only approximate if there are small random shocks A_t and υ_t. With shocks, steady states become stochastic and the concept of convergence or divergence is in general probabilistic. One can numerically construct a domain of attraction for example by requiring convergence with probability one. (Such a domain would be inside Ω.) Then convergence to

[12] Figure 5 below shows a numerical illustration of the phase diagram and the domain of attraction of (π^*, y^*). The diagram is limited to an area that includes steady states (π^*, y^*) and (π_L, y_L).

[13] See e.g. Benhabib, Evans, and Honkapohja (2014) and the references therein.

[14] For more details, see Evans, Honkapohja, and Mitra (2020).

the target steady state occurs only with positive probability if the economy moves slightly across the boundary of a stochastic domain of attraction.[15]

5 Effects of Shocks from Pandemic

The data in Section 1 clearly indicates that the Covid pandemic has resulted in major shocks to the economy and the episode is likely to continue for some time. Our stylized model, summarized in Sections 2 - 4, is now used to uncover some central implications of a pandemic on an economy recovering from stagnation.[16] The model is very stylized and hence does not account for many of special aspects of a recession caused by the pandemic. Nevertheless the analysis provides some basic lessons about consequences of pandemic shocks in an economy in stagnation.

As a starting point it is assumed that the economy is in the middle of a recovery from stagnation.[17] Formally, according to the model the economy is inside the domain of attraction but away from the target steady state, i.e. in Figure 4 the current state is point above the curve SS but located South-West of the target steady state.

One shock at the onset of a pandemic is that economic expectations about future output and inflation abruptly shift in negative direction and there is also increased uncertainty about the future development of the economy. Such shocks are also part of a usual recession. The model is designed for analysis of the effects of a negative shift to output expectations i.e. a downward shift in the perceived demand curve faced by the producers.

Importantly, a pandemic also influences the supply side of the economy. Production activities face new difficulties of various type, including the health of workers and even lock-down of the sector. The supply-side effects probably differ between sectors with especially the service sector being badly hit. However, contractionary development of the "directly affected" sector negatively influences also the "unaffected" sector, so the economy overall is hit by the pandemic.[18] Our model is a one-good economy so the effects are proxied by a negative shock to total productivity A_t.

We thus consider two effects of the pandemic:

[15] Probability of convergence goes to zero if one moves further outside the boundary. For examples see Evans, Honkapohja, and Mitra (2020).

[16] As already noted, the model we use is presented in detail in Evans, Honkapohja, and Mitra (2020).

[17] We set $\pi_0^e = 1.000375$, i.e. expected inflation of 1.5 percent in annual terms, and $y_0^e = 0.9985$, i.e. 2.3 percent below y^* in terms of two-year equivalents computed in the cited paper.

[18] The transmission betwen sectors is emphasized e.g. in Blanchard (2020) and Guerrieri, Lorenzoni, Straub, and Werning (2020).

(i) There is increased pessimism. In the model the pessimistic shock is formally represented by a negative shock to output expectations y_0^e that describe current forecasts of the economy about the medium to long term.
(ii) Production technology receives a negative shock and the economy becomes less efficient. This is formally represented by a negative shock to total factor productivity A_t, $t = 0, ...$

We discuss the consequences of (i) and (ii) separately.

5.1 Shock to output expectations

The effects of a negative expectations shock about future aggregate demand are fairly familiar, see Evans, Honkapohja, and Mitra (2020) for details in the current model. We summarize the details briefly here.

Assume that the economy is recovering from stagnation and in terms of Figure 4 is inside the domain of attraction Ω and moving on a convergent path toward the target steady state. Then a pessimistic shock i.e. negative shock to expectations of aggregate demand y_0^e hits the economy. (For simplicity, assume that inflation expectations π_0^e remain unchanged.) [19]

Note that the direction of the shock is qualitatively in line with the data in Figures 1 and 2. If the shock is small, the economy remains inside Ω and continues on a convergent path to the target steady state. In contrast, when the shock is sufficiently big, it displaces the economy outside Ω and the economy begins to move towards stagnation if fiscal policy remains unchanged. Correcting this divergence requires expansionary fiscal policy (increased government spending) which needs to be sufficiently large.[20]

5.2 Shock to productivity

Assume that the state of the economy before the shock is the same as in the preceding section, i.e. it is in the middle of an ongoing recovery from stagnation and is moving toward the target steady state. Then a supply side shock hits the economy, as in the current period total factor productivity A_t declines permanently (or for many periods).

[19] Formally, in the model y_0^e shifts to 0.9975, which is about one percentage points decline in terms of the equivalents mentioned in the preceding footnote. From time $t = 1$ onward expectations start to adjust according to learning. One could assume that shock to y_0^e affects expectations for more periods.

[20] In the model there are also cases where the probability of convergence to target steady state lies between zero and one. For examples, see Evans, Honkapohja, and Mitra (2020).

The effects of this shock are more complex than a shock to expectations, because a shift in aggregate productivity changes the structure of the economy. The shock thus moves the target steady state. Variations in A_t directly imply shifts in the Phillips (or aggregate supply) curve, see the variable A^e_{t+s} in (3). There will also be a shift in the monetary policy rule (1) if the interest rate rule depends on the output gap. Looking at Figure 4, a negative productivity shock shifts the domain of attraction Ω downward and also changes the shape of Ω somewhat. This takes place as, quite naturally, lower productivity decreases the steady state level of output y^*.[21] The shape of Ω usually changes because of the nonlinearities. Figure 5 illustrates numerically the movement of the domain of attraction after a shift in A_0.[22]

FIGURE 5 ABOUT HERE

The dynamics after the shock depend the magnitude of the negative productivity shock. If the magnitude of the permanent shock is small, then the economy continues to converge towards the new steady state with a lower value for output. However, after a larger productivity shock, the economy may start to move away from the post-shock target steady state.[23]

The shifting structure of the economy from the change in productivity can create surprising outcomes. It can happen that after a negative productivity shock convergence back to (post-shock) target steady state from an initial condition $(\hat{\pi}^e_0, \hat{y}^e_0) \approx (\pi^*, y^*)$ before shock is more fragile than from different initial condition $(\tilde{\pi}^e_0, \tilde{y}^e_0)$ that is below (π^*, y^*) in both components. The productivity shock causes a shift of the target steady state and the domain of attraction shifts downwards as illustrated in Figure 5. After the shift it is possible that $(\tilde{\pi}^e_0, \tilde{y}^e_0)$ is better situated than $(\hat{\pi}^e_0, \hat{y}^e_0)$ for convergence to the post-shock steady state.

Another surprising result is that a negative productivity shock can sometimes stabilize an unstable situation for the economy. To see this assume that the initial pre-shock condition (π^e_0, y^e_0) is slightly below the (pre-shock) middle steady state, i.e. $\pi^e_0 < \pi_L$, $y^e_0 < y_L$ (so if no shock occurs the economy is beginning to descend to stagnation) and then a negative productivity shock occurs. One possibility is that the economy continues to converge toward stagnation steady state. However, depending on the magnitude of the shock, it is also possible that the economy begins

[21] Steady state inflation π^* remains unchanged as it is part of the monetary policy framework.

[22] The shift in A_0 is from 1.113 to 1.11. The parameter values in the model are otherwise those used in Evans, Honkapohja, and Mitra (2020).

[23] The pre-shock value is $A = 1.113$. For a permanent shock and given starting point (π^*, y^*) at the steady state, there is convergence for $A = 1.112$ and divergence for $A = 1.111$.

to converge toward the post-shock target steady state. This is the case if (π_0^e, y_0^e) is inside post-shock domain of attraction Ω.

The analysis just presented can be extended to consider the more realistic case of a negative covid productivity shock that has finite duration. In this case the general flavor of the results remains similar to results with a permanent shock but there are some modifications. [24]

6 Concluding Remarks

We have studied the impacts of demand and supply shocks from the appearance of a pandemic on an economy in stagnation or recovering from stagnation. In the model a pandemic is thought to result in (i) negative demand shocks to aggregate output and inflation and (ii) negative supply shocks to productivity in the economy. Such shocks usually add to stagnation pressures. As regards policy, usual fiscal policy in the form of increased government spending on consumption goods is the right kind of response to a demand shock due to increased pessimism in the form of decline in output expectations. In contrast, a negative productivity shock is a supply side phenomenon and would require different response that offsets the decline in aggregate productivity due to the pandemic. One natural possibility would be increased public investment that raises productivity in the economy and thereby offsets the shock to A_t. This case was not studied as our simple model excludes private and public investment.

It should be emphasized that our analysis is very much a first approach. In particular, it is assumed that the economic impact of an epidemic comes about only through exogenous (permanent or persistent) shocks. Dynamic interactions of the pandemic with the economy, e.g. on the labor force through health, are excluded from the model.[25]

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[24] For instance, a negative shock to A_0 with finite duration may result in convergence towards stagnation even though a permanent shock of the same magnitude would deliver convergence to (post-shock) target steady state. Simulations are available on request.

[25] See e.g. Eichenbaum, Rebelo, and Trabandt (2020) for a model with these effects.

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Figures and Tables

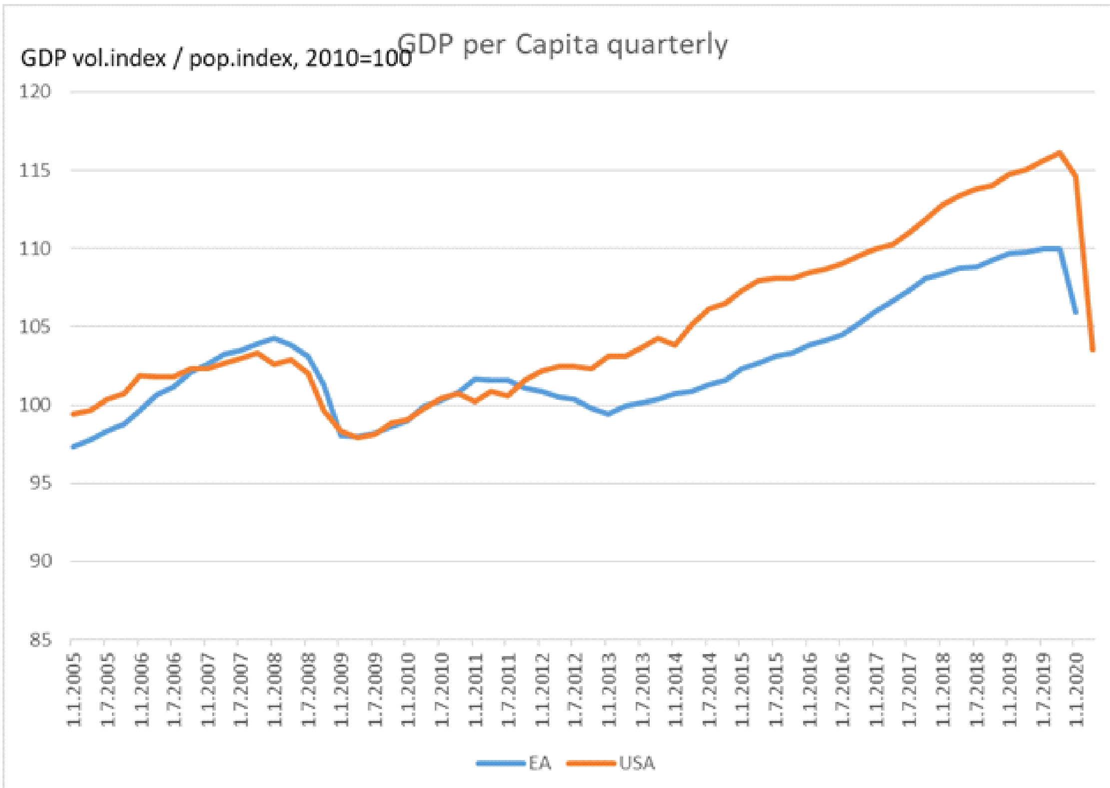

Figure 1: real GDP per capita, US and Euro area

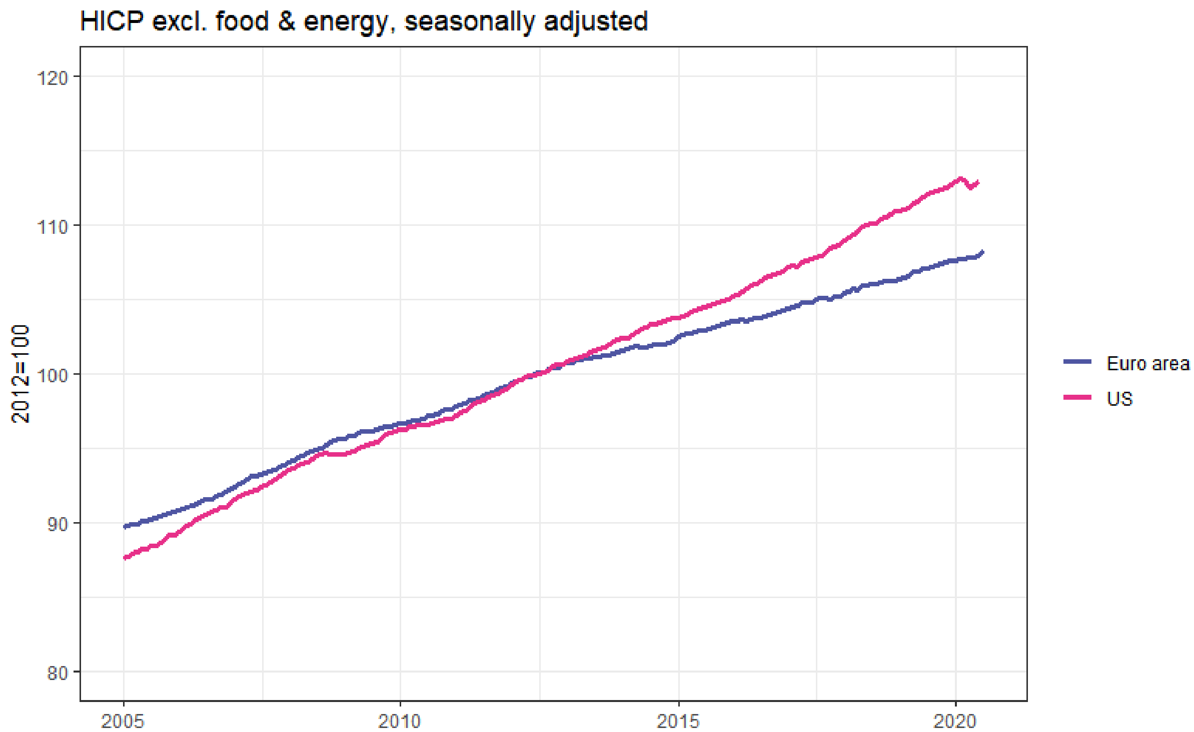

Figure 2: US and Euro area monthly consumer prices, excluding food and energy in 2005-2020.

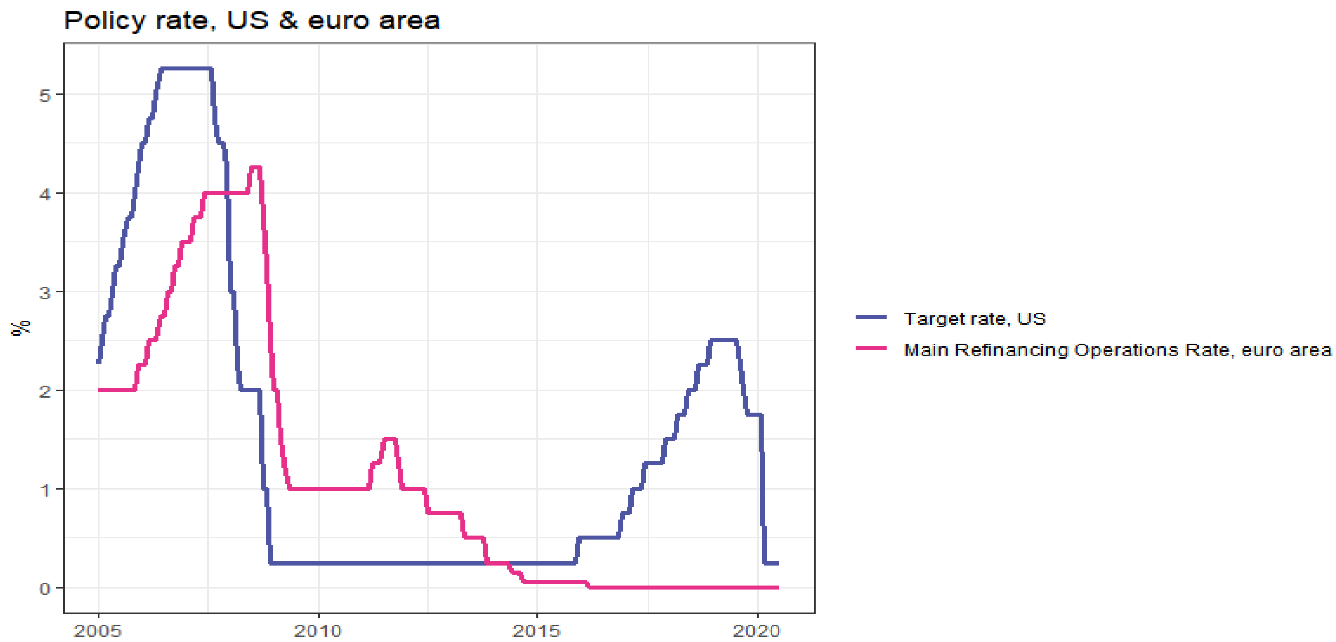

Figure 3: Policy interest rates of the US Federal Reserve and the ECB fro m2005 to present.

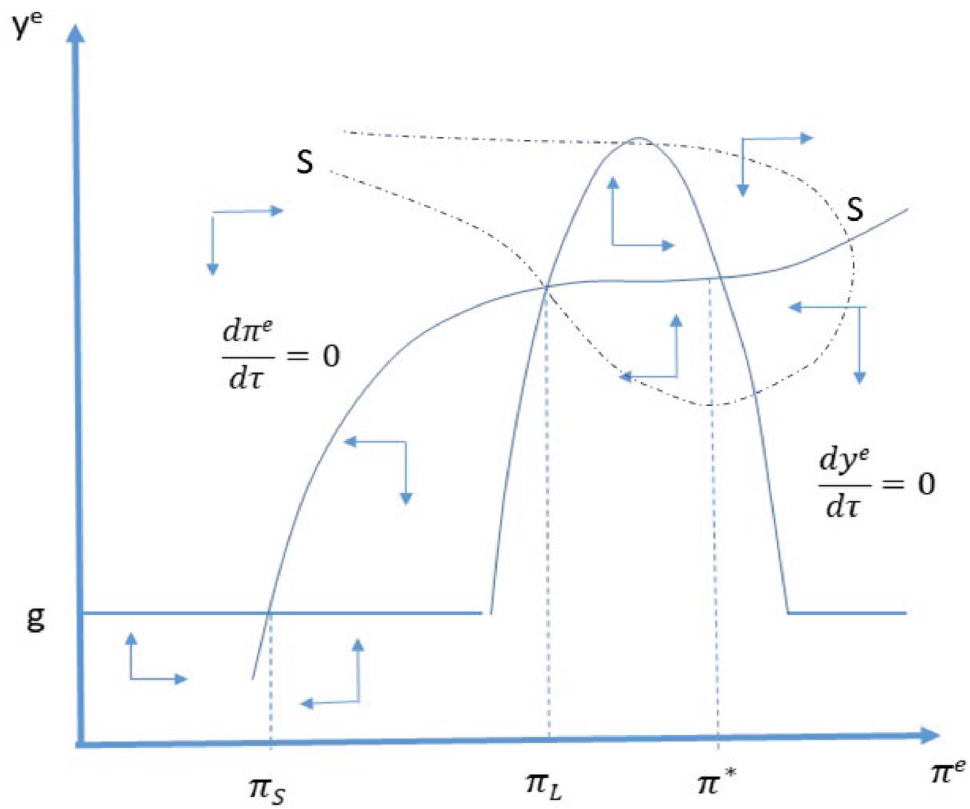

Figure 4: Global E-stability dynamics with curve SS giving the boundary of Ω.

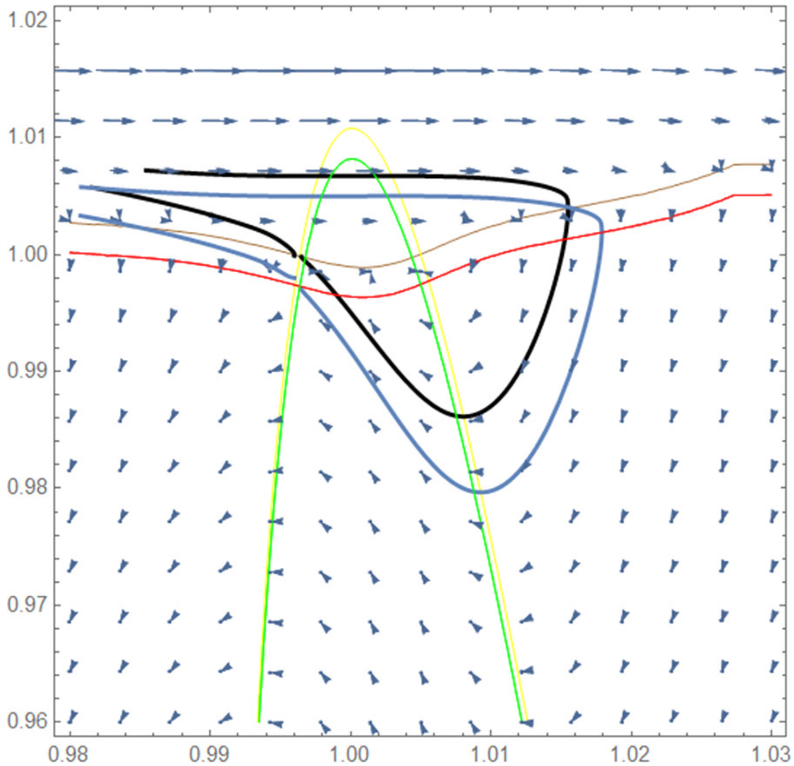

Figure 5: Effect of productivity decline on the phase diagram.