Abstract

The social cost of climate change is unknown and may remain so for the next 50-100 years. Does the value of waiting for better information matter for policies? For a quantitative answer, we develop a tractable stochastic climate-economy model with a hidden-state process for catastrophic impacts. The value of waiting is negligible: persistent potential impacts imply slowly changing beliefs about their arrival. The optimal policies lead to “decarbonization”, even without experienced economic climate-change impacts. Taking a dataset of estimates for the social cost as a representation of current beliefs, our quantitative assessment shows that the rising income is the main driver for the carbon price for the coming century. The carbon price should grow faster than the economy as long as climate warming is not enough for generating impacts that are informative about the true social cost.

(JEL classification: H43; H41; D61; D91; Q54; E21. Keywords: carbon price, learning, climate change)
1 Introduction

A price for carbon measures the social cost of releasing a unit of carbon dioxide to the atmosphere, based on expected climate-change impacts. However, there is little or no quantitative information on the impacts of persistent climate change on our economies, although there is extensive research on what such impacts might be. The social cost of carbon is based on beliefs about impacts that will be updated when the “climate experiment” generates actual, potentially catastrophic, impacts. But this can take a long period of time; the past century of carbon emissions has not yet led to precise estimates, and another 50-100 years may pass without additional hard evidence on the ultimate consequences of current emissions. Over such long periods, policies on global warming have to deal with fundamental changes in the economy as much as with forthcoming information on how seriously the economy will be impacted.

Economic growth stands out as a major driver of global change when considering a period such as the next 50-100 years. For the coming century, global income is expected to grow by multiple factors, in part due to the rise of the middle class in major emerging economies. The US government has recently developed estimates for the carbon price, for regulatory purposes, assuming that the global GDP increases by a factor that varies between five and seven in this time-span (see Greenstone et al., 2013). The future economy grown five to seven times bigger, and with almost a century of extra climate-change experience, prices emissions differently — but how exactly?

Increasing incomes lead to higher economic losses, and thereby drive a gradual tightening of policies over time, the “climate policy ramp” (Nordhaus 2007; Golosov et al. 2014). Further support for the policy ramp is given by expectations that mitigation options become cheaper, and that climate-change losses are increasing more than proportional.

1See IPCC (2014) for a survey on methods and results. There is a growing empirical literature on how climate impacts various sectors of the economy (e.g., Deschenes and Greenstone, 2007, and Schlenker and Roberts, 2009, Dell, Jones, and Olken, 2012).

2See, for example, the IPCC Special Report on Emissions Scenarios (2000), U.S. Climate Change Science Program (2007), Stanford Energy Modeling Forum (for example, in Weyant et al. 2006).

3There are other arguments such as green technological change for not following gradualism but rather a jump-start in emissions pricing (van der Zwaan et al. 2002; Gerlagh, Kverndokk and Rosendahl 2009; and Acemoglu, Aghion, Bursztyn, and Hemous, 2012).
portionally with temperature change (Wigley et al., 1996). Additionally, when economic
growth levels off, capital returns will diminish (Piketty and Zucman 2014), and thus the
relative returns of the climate investments increase.

But the policy ramp, as such, does not describe carbon prices for the situation where
climate impacts are fundamentally unknown, including the possibility of a catastrophic
climate outcome. Consider, for example, an economy where income growth progresses,
but without verifiable economic climate-change impacts, over periods such as the next
50-100 years. Is the lack of evidence for economic losses from climate change enough for
lowering the perception of the social cost?

We develop a climate-economy model with closed-form policy rules for pricing uncer-
tain high-consequent events. The policy rules are sensitive to past experience; they show
how economic growth supports a policy ramp while the absence of substantial climate
impacts leads to increasing optimism putting a downward pressure on carbon prices.
But if the current level of climate change cannot generate experienced impacts that are
substantial enough for learning the long-run cost, we find that the expected social cost
of carbon grows faster than the economy.

In a quantitative assessment, we address the question if there can, in principle, be
a case for increasing optimism that reverses the ramp implied by economic growth. To
this end, we take a dataset of existing estimates for the social cost, collected by Tol
(2009), as a representation of current beliefs. We find that one has to rule out severe
climate impacts by orders of magnitude faster than what is implied by the scenario in
our explorative calibration. It matters very little for the policy ramp that the existence
of impacts has to be learned over time. Quantitatively, the result confirms that the
option value of waiting for better impact information is negligible — robust information
is unlikely to arrive any time soon. Thereby, the determinants of the carbon price path are
fundamentally economic by nature; that is, the same that underly the ramping arguments
in the mainstream climate-economy literature.\footnote{Among climate researchers, the delays in learning the impacts are widely accepted – yet, economist
have not communicated the meaning of delays for the policy ramp. Roe and Baker establish (2007)
that, because of positive feedback mechanisms of the climate system, it is unlikely that we will better
understand the temperature sensitivity to emissions in the near future. The economic literature modeling
the learning of climate impacts has almost exclusively focused on the structural uncertainties of the
climate system, including those related to the climate sensitivity (Kelly and Kolstad 1999; Leach 2007;
Kelly and Tan 2013) and to unknown thresholds leading to tipping points (Lemoine and Traeger 2014;
Cai, Judd, and Lontzek, 2013). For many economists, such climate uncertainties and the implied low-
probability but high-consequence events, which cannot be ruled out by new information any time soon,}
not minor: as we show, the extreme persistence of beliefs about ultimate impacts leads to optimal full decarbonization of the economy, even without actual experience of such climate-change impacts.

These results provide a bridge between the climate-economy models (Integrated Assessment Models, IAMs) based on middle-of-the-road impact estimates and their critics expressing concerns that these models do not cover “unknowns” that should be the main reason for having a carbon price (Pindyck 2013). We reconcile the views in a model supporting a carbon price ramp, not based on smooth and moderate climate change damages, but through a description of uncertain high-consequent events and belief updating. The model is detailed on the key climate-economy interactions and it is analytic; it avoids the “black-box” nature of some of the numerical IAMs.

We build on the Brock-Mirman model (1972) for the climate-economy interactions, following Golosov, Hassler, Krusell, and Tsyvinski (2014); however, we introduce climate change differently through a hidden state that determines whether a negative productivity shock can hit the economy in the future. Beliefs on the hidden state allows including heterogeneity of views, and the structural interpretation of the social cost data. Moreover, we adopt a richer emissions-temperature response that includes the necessary delays between emissions and potential impacts, which have substantial implications for the quantitative assessment.

This workhorse model in analytical macroeconomics lends itself to a relatively transparent, though not a standard quantitative assessment of the climate-economy interaction. There are no objective observable data that can be used to calibrate a model on information generation and beliefs updating for catastrophe occurrences. Instead, have become the prime argument for having a price for carbon (Weitzman, 2009, 2011, 2013). See also Lange and Treich (2008), and Heal and Millner (2013) for surveys on uncertainties in climate-change economics.

Most evaluations of the social cost of carbon build on a set of middle-of-the-road assumptions on climate change impacts, commonly expressed in terms of GDP losses, and then use climate-economy models such as DICE, FUND, or PAGE (see Greenstone, Kopits, and Wolverton (2013) for a succinct description and references) that combine the impact assumptions with background scenarios to obtain a monetized value for the social cost.

By the nature of our quantitative exercise, we rule out “tail events”. The supporting potential high-damage climate event that justifies the estimated initial carbon price is equivalent to a GDP-loss of about 10 per cent at temperatures that are 3 degrees Celsius above the pre-industrial level. Such an event is economically significant but not a “tail event” in the sense of Weitzman (2009) where policies become undefined since, effectively, it is not possible to transfer wealth to the high consequence events; see, for example, Nordhaus (2010) and Millner (2013) for discussion.
we “reverse engineer” our analytical model, such that it quantitatively reproduces the distribution of carbon prices found in the literature.

We intentionally devise a conservative test against the policy ramp through a “no news is good news” scenario: the absence of damages leads to more optimistic assessments over time. The information acquisition is allowed to work against the upward pressure coming from income growth. This avoids, on purpose, a structure that supports tightening climate policies from increasing probabilities for climate tipping points (Lemoine and Traeger 2014; Cai, Judd, and Lontzek, 2013). The structural interpretation of the existing carbon price distribution allows us to assess how quickly one would need to be able to rule out severe impacts to overturn the climate policy ramp.

The paper is structured as follows. In Section 2, we first explain the basic planning problem and the natural-science part; that is, the emissions-temperature response that follows from the description of the global carbon cycle. We set up the model to allow for two conceptually very different views on learning. The first (binary) view on learning is tailored to catastrophic events. The expected social costs depend purely on beliefs without actual experience, until such experience, if ever, occurs. The second (continuous) view resembles the, more standard, smooth learning. Climate change generates continuously real losses and thus information about future losses. We cover both types of learning in one common set up; the explicit policy rules take a very similar form in both situations. However, the main body of the paper, including the quantitative assessment focuses on the first type of learning. In Section 3, we consider the binary learning model, its calibration, and the quantitative assessment. In Section 4, we introduce smooth learning, and derive the long-run carbon price distributions. These underscore our main message: the long-run carbon price determinants are fundamentally economic by nature. Section 5 concludes. The online supplementary file contains a program for reproducing the graphs in the text.

2 The climate-economy model

2.1 The basic setting

We consider a climate-economy planning problem where production possibilities at time $t$ depend on capital $k_t$ inherited, and potentially also on the full history of carbon input

\footnote{Follow the link \url{https://www.dropbox.com/sh/7meos655j14jh5p/_d1r8X_PHI}}
use,

\[ s_t = (z_0, ..., z_{t-1}). \]

Given \( k_t \) and history \( s_t \) at time period \( t \), consumption, \( c_t \), and carbon inputs, \( z_t \), are chosen to maximize the expected discounted utility

\[
\max \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau u_{t+\tau}
\]

where \( 0 < \delta < 1 \) is the discount factor and \( u_{t+\tau} \) is the periodic utility, specified below. The chosen allocations must satisfy

\[
c_t + k_{t+1} = y_t,
\]

with \( y_t = f_t(k_t, s_t, z_t) \) denoting the output at time \( t \). Capital thus fully depreciates in a period; on the other hand, a unit of time in our quantitative assessment will be a decade, which partly offsets the assumption. Losses due to climate change depend on the history of emissions \( s_t \) through variable \( D_t \) that is a measure of the global mean temperature increase above the pre-industrial levels at time \( t \). We assume that this measure is a function of history \( s_t \),

\[
D_t = \sum_{\tau=1}^{t} R(\tau)z_{t-\tau}
\]

where the weights \( R(\tau) \) define the “emissions-temperature response”. That is, current emissions \( z_t \) affect temperatures at some later time \( t + \tau \) according to a known response function \( R(\tau) \):

\[
\frac{dD_{t+\tau}}{dz_t} = R(\tau) > 0.
\]

Below, in equation [8], we provide a parametric form for the response function, connecting to the fundamentals of climate problem. The key characteristic of \( R(\tau) \) is the considerable delay of the response following an impulse of emissions; it has a non-linear shape peaking several decades after the date of the emissions, and a fat tail of almost permanent impacts. Simplistically, there is no uncertainty about \( R(\tau) \); the response serves the purpose of introducing delays to the potential impacts on the economy that, in turn, will be uncertain. As will be seen, the structure of temperature delays and persistence will importantly shape, for example, the calibrated learning rate for impacts.

Output is given by production function

\[
y_t = k^\alpha_t A_t(z_t) \exp(-\Delta_{y,t}D_t),
\]
where $0 < \alpha < 1$ and the contribution of carbon inputs $z_t$ enter through the function $A_t(z_t)$ that captures the energy sector of the economy. Quantitative analysis captures the total factor productivity development of the economy as well as the energy sector through $A_t$ but, since the analytical results do not require a specific form for $A_t$, we postpone the detailed discussion of $A_t$ to Section 3.5.4 and the extension to stochastic total factor productivity to Section 4. Here, we merely assume that carbon input $z_t$ has a positive but diminishing marginal product.

Losses from the temperature increase arise potentially from two sources. First, they can lead to reduced output, through the negative productivity impact in (5), as in most applied climate-economy models (e.g., Nordhaus, 2008). This impact depends on the full history of emissions, determining current temperature $D_t$ through (4), and damage coefficient $\Delta y,t \geq 0$ that is a stochastic variable. Second, the temperature may have a direct impact on periodic utility that we define as

$$u_t = u(c_t) - \Delta u,t D_t,$$

where $u(c_t) = \ln(c_t)$, $\Delta u,t \geq 0$. The two potential sources for losses capture the climate-change unknowns: while the current and past losses are known, the sequence of future impacts

$$\{\Delta y,t+\tau, \Delta u,t+\tau\}_{\tau>0}$$

is not known to the climate policy maker. In this paper, we consider two distinct processes generating the future climate-change impacts, allowing the decision maker to form expectations $E_t\{\Delta y,t+\tau|\Omega_t\}$ and $E_t\{\Delta u,t+\tau|\Omega_t\}$, where $\Omega_t$ is the current information set. In the first one, the economy starts with no experienced losses but may irreversibly enter a climate-economy state where potentially catastrophic damages occur. Thus, the decision-maker learns by (not) observing damages, which allows updating the beliefs on the ultimate arrival of such damages. In the second specification, the climate generates “experience” and thus evidence on a continual basis through events such as hurricanes, hot-summer spells, or perhaps a long-period of stable climatic conditions; whether experienced events are due to climate change or within the normal variation is initially a matter of beliefs. In both cases, climate change, as captured by the current temperature and the dependence of future temperatures on historical emissions in (4), is known, but

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8Note that we do not impose an upper limit for the total use of carbon inputs, implying that the exhaustible-resource nature of such inputs is ignored. The assumption is motivated by the size of carbon deposits in the form of coal that exceeds absorptive of capacity of the atmosphere. See Gerlagh (2011) and van der Ploeg and Withagen (2012).
the distribution generating future economic losses in (7) is unknown. Below we introduce first the temperature response, and then the two approaches for learning the impacts.

2.2 Climate dynamics

The temperature response to emissions is a key determinant of the expected present-value utility impacts of the current emissions, that is, the social cost. For tractable policies, we build on a closed-form for $R(\tau)$ that is derived in Gerlagh and Liski (2013), Theorem 1.

Remark 1 Consider a carbon diffusion process, described by a set of impulse-responses $I$, with fraction $0 < a_i < 0$ of emissions having decay rate $0 \leq \eta_i < 1$, $i \in I$. For temperature sensitivity $\pi$ and adjustment speed $\varepsilon$, the impact of emissions at time $t$ on temperatures at time $t + \tau$ is

$$\frac{dD_{t+\tau}}{dz_t} = R(\tau) = \sum_{i \in I} a_i \pi \varepsilon \frac{(1 - \eta_i)^\tau - (1 - \varepsilon)^\tau}{\varepsilon - \eta_i} > 0. \quad (8)$$

To explain this result, we outline first the two main determinants of the response: the carbon cycle and the relationship between carbon concentrations and temperatures. The carbon cycle refers to a diffusion process of carbon between reservoirs of carbon, such as those in the atmosphere, oceans and biosphere. Obviously, the atmospheric reservoir is the one relevant for climate warming but the other reservoirs are relevant for the delays and persistencies of changes in the atmospheric stock. Assuming a linear diffusion, the system can be de-coupled by eliminating interactions between the reservoirs, leading to an isomorphic system of separable impulse-responses for carbon stocks (Maier-Reimer and Hasselman 1987). The shares and decay rates have intuitive meanings, discussed below, and they follow from the physical description of the system of carbon reservoirs.

The carbon cycle is relatively well understood in natural sciences but the relationship between temperatures and carbon concentrations is fundamentally uncertain (see, for example, Roe and Baker, 2007). Acknowledging these complications, we note that economic impacts introduce yet another layer of fundamental uncertainty; we focus on this uncertainty and make the following simplistic assumptions on the determinants of the climate equilibrium. Emissions $z_t$ increase the atmospheric $CO_2$ stock, through the

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9The true diffusion process is non-linear (Joos et al. 2013). The linear representation is an approximation.
carbon cycle, and there is a linear relationship between the steady state atmospheric $CO_2$ stock and the steady state level of $D_t$. This relationship is captured by parameter $\pi$: a one-unit increase in the steady-state atmospheric $CO_2$ stock leads to a $\pi$-unit increase in the steady-state level of $D_t$. Outside steady state, there is a delay in the effect from concentrations to temperatures, and this delay is captured by parameter $0 < \varepsilon < 1$: a one-unit increase in emissions increases the next period $CO_2$ stocks one-to-one but the direct temperature increase is only $\varepsilon \pi$-units.

Parameter $\eta_i$ captures, for example, the carbon uptake from the atmosphere by forests and other biomass, and oceans. The term $(1 - \eta_i)^\tau$ measures how much of carbon $z_t$ under decay $i$ still lives after $\tau$ periods, and the term $-(1 - \varepsilon)^\tau$ captures the slow temperature adjustment. The limiting cases can be helpful. Consider one $CO_2$ reservoir. If atmospheric carbon-dioxide does not depreciate at all, $\eta = 0$, then the temperature slowly converges at speed $\varepsilon$ to the long-run equilibrium climate sensitivity $\pi$, giving $R(\tau) = \pi[1 - (1 - \varepsilon)^\tau]$. If atmospheric carbon-dioxide depreciates fully, $\eta = 1$, the temperature immediately adjusts to $\pi \varepsilon$, and then slowly converges to zero, $R(\tau) = \pi \varepsilon (1 - \varepsilon)^{\tau-1}$. If temperature adjustment is immediate, $\varepsilon = 1$, then the temperature response function directly follows the carbon-dioxide depreciation $R(\tau) = \pi (1 - \eta)^{\tau-1}$. If temperature adjustment is absent, $\varepsilon = 0$, there is no response, $R(\tau) = 0$.

![Figure 1: Emissions-damage response. The path depicts the output loss associated with $1TtCO_2$ impulse of carbon at time $t = 0$ for $\Delta_y = 1$ and $\pi = .0156$.](image)

When multiplying temperature measure $D_t$ by given output-loss coefficient $\Delta_y > 0$, we can interpret the emissions-temperature response as an emissions-damage response.
Fig. 1 shows the life path of damages (percentage of total output) caused by an impulse of one Teraton of Carbon \([\text{Tt}CO_2]\) in the first period.\(^{10}\) The output loss is thus measured per \(\text{Tt}CO_2\), and it equals \(1 - \exp(-\Delta_y R(\tau))\), \(\tau\) periods after the impulse. The non-monotonicity of the response, as depicted in Fig. 1, captures well the climate impact dynamics, for example, in DICE-2007 (Nordhaus, 2008).

The physical data on carbon emissions, stocks in various reservoirs, and the observed concentration developments can be used to calibrate a three-reservoir carbon cycle representation; we choose the following emission shares and depreciation factors per decade:\(^{11}\)

\[
a = (0.163, 0.184, 0.449) \\
\eta = (0, 0.074, 0.470).
\]

Thus, about 16 per cent of carbon emissions does not depreciate while about 45 per cent has a half-time of one decade. We assume \(\varepsilon = 0.183\) per decade, implying a global temperature adjustment speed of 2 per cent per year. Normalizing the output loss parameter at unity, \(\Delta_y = 1\), and setting \(\pi = 0.0156\) [per \(\text{Tt}CO_2\), see Gerlagh and Liski (2013)] is consistent with the Nordhaus (2008) baseline where a temperature rise of 3 degrees Celsius leads to about 2.7 per cent loss of output.\(^{12}\) These quantitative choices parametrize the emissions-temperature response that is depicted in Figure 1. In the calibration below, we allow \(\Delta_y\) to be determined by the distribution for the carbon prices obtained from previous studies; throughout, \(\Delta_y = 1\) refers to the Nordhaus’ baseline.

### 3 Learning from no news

Consider now an economy where the temperature increase is observed but there may be a long waiting time for experiencing the impacts. There are two climate-economy states, \(I_t \in \{0, 1\}\). If \(I_t = 0\), no damages have been experienced by \(t\). If \(I_t = 1\), damages have appeared, and once \(I_t = 1\), then \(I_{t+\tau} = 1\) for all \(\tau \geq 0\). The damage coefficient at time

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\(^{10}\)One \(\text{Tt}CO_2\) equals about 25 years of global \(CO_2\) emissions at current levels (40 Gt\(CO_2/\text{yr.}\)).

\(^{11}\)Some fraction of emissions depreciates within one decade from the atmosphere, and therefore the shares \(a_i\) do not sum to unity. The choices here are based on Gerlagh and Liski (2013) but similar representative numbers can be found in the scientific literature; see, e.g., Maier-Reimer and Hasselman (1987).

\(^{12}\)To clarify the units, the damages are measured per Teraton of \(CO_2\) [Tton\(CO_2\)], and the 3 degrees Celsius rise follows from doubling the \(CO_2\) stock. We have chosen the value of \(\pi\) such that the normalization \(\Delta_y = 1\) gives the Nordhaus case. For this reason, the interpretation of \(\pi\) is “climate damage sensitivity” rather than “climate sensitivity”.

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$t$ for output, affecting production in (5), is $\Delta_y,t = \Delta_y I_t$, where $\Delta_y > 0$ is a constant, independent of time. Similarly, the direct utility loss, entering (6), is $\Delta_u,t = \Delta_u I_t$.

The economic problem defined through (1)-(6) is essentially the Brock-Mirman (1972) consumption-choice model which has a well-explored structure, apart from the climate-economy interactions. The state vector is $(k_t, s_t, \Omega_t)$, where the information set includes $I_t$ and the current belief, defined shortly. Because of the log-utility for consumption, full capital depreciation in one period, and Cobb-Douglas capital contribution, share

$$g = \alpha \delta$$

of the gross output will be saved; the dynamic programming arguments leading to this policy are well known in analytical macro-economics (Sargent, 1987). Moreover, given the exponential form for the potential output loss and the linearity of the direct utility loss, welfare in (11) will be separable with respect to the contribution of $k_t$ and $s_t$. Thus, the climate policy analysis can be conducted by taking savings $g$ as given and by tracking the direct utility impacts of the potential loss from climate change. It proves useful to aggregate both the potential output and direct utility losses into one measure:

Remark 2 For $I_t = 1$, the present-value loss of utils from marginal climate change at time $t$ is

$$\Delta \equiv - \sum_{\tau=0}^{\infty} \delta^\tau \frac{du_{t+\tau}}{dD_t} = \Delta_u + \frac{\Delta_y}{1-g}. \tag{10}$$

Thus, output and direct utility losses are comparable in terms of their effect on utility; for convenience, we will use $\Delta$ as an aggregate measure of both losses. In addition, note that here we consider a change in $D_t$ while keeping the temperature in other periods constant. For the proof of the remark, consider the effect of temperature $D_{t+\tau}$ on utility in period $t + \tau$ when $I_t = 1$ (climate impacts have arrived). Recall that the consumption utility is $\ln(c_{t+\tau}) = \ln((1-g)y_{t+\tau}) = \ln(1-g) + \ln(y_{t+\tau})$ so that, through the exponential output loss, the consumption utility loss is given by $\partial \ln(c_{t+\tau})/\partial D_{t+\tau} = -\Delta_y$. As there is also the direct utility loss, captured by $\Delta_u$ in (6), the full loss in utils at $t + \tau$ is

$$- \frac{du_{t+\tau}}{dD_{t+\tau}} = \Delta_y + \Delta_u.$$

But, part $g$ of the output loss at $t + \tau$ also propagates through savings to period $t + \tau + 1$ and further to periods $t + \tau + n$ with $n > 0$, so that the full loss of utils, discounted to time $t$ and denoted by $\Delta$, is given by (10).

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13See Golosov et al. (2014) or Gerlagh and Liski (2013).
3.1 Beliefs

The hazard rate for damages, denoted as $p$, is the probability that damages start and $I_t = 0$ moves to $I_{t+1} = 1$. The hazard rate is a given constant for each period, but unknown to the policy maker. We assume that $p$ has a discrete prior distribution: it can either take value $p = 0$ or $p = \lambda$. The hazard rate can depend on the degree of climate change as measured by $D_t$, for example, so that only for periods where $D_t > \overline{D} \geq 0$ the state can switch. We postpone this extension in Section 3.4 and assume now learning in all periods by setting $D = 0$.\footnote{For example, $\overline{D}$ can correspond to 2-degrees Celsius warming, but since we have little information about the learning thresholds, we will set $\overline{D} = 0$ in the calibration. The solution of the model can be easily extended to the case of different temperature brackets, all having different hazard rates.}

There is no prior climate experiment; we do not know the value of $p$, but we assume a subjective prior probability $\mu_0 > 0$ for a positive hazard rate, $p = \lambda$. The probability for eventual climate impacts satisfy:

$$1 - \mu_0 = \Pr(\lim_{t \to \infty} I_t = 0) = \Pr(p = 0)$$

$$\mu_0 = \Pr(\lim_{t \to \infty} I_t = 1) = \Pr(p = \lambda > 0).$$

Let $\mu_t$ denote the posterior probability that $p = \lambda$, at time $t$, conditional on no impacts having yet occurred by time $t$, $I_t = 0$. Each period where $D_t > \overline{D} = 0$, but where no damages have appeared so far, $I_t = 0$, climate change runs an experiment. If the outcome is $I_{t+1} = 1$, which happens with probability $\mu_t \lambda > 0$, we have learned that $p = \lambda$, so $\mu_{t+1} = 1$. If the outcome is $I_{t+1} = 0$, we have not learned the state of nature with certainty, but the beliefs are updated to $\mu_{t+1}$. We can write the Bayesian updating rule as\footnote{Note that $\Pr(p = \lambda | I_t = 0) \times \Pr(I_t = 0) = \Pr(p = \lambda \cap I_t = 0)$. The probability that there has been no news by time $t$ is $\Pr(I_t = 0) = \mu_0(1 - \lambda)^t + 1 - \mu_0$. The probability that there has been no news by time $t$ and that $p = \lambda$ is $\Pr(p = \lambda \cap I_t = 0) = \mu_0(1 - \lambda)^t$. Combining gives the equation.}

$$\mu_t = \Pr(p = \lambda | I_t = 0)$$

$$= \frac{\mu_0(1 - \lambda)^t}{\mu_0(1 - \lambda)^t + 1 - \mu_0}$$

which is the probability that climate change impacts will ultimately arrive even though such damages have not been experienced by time $t$. Note that $\mu_t$ declines over time: “no news is good news”; the assessment of the distribution for damages becomes more optimistic over time.\footnote{One could argue that impacts must ultimately arrive for a sufficiently severe climate change. While...}
stochastic process for damages, and the size of damages, respectively. While these items items enter parametrically the climate decision problem, the values calibrated to best match the carbon price data depend on the overall model structure; for example, the increasing the persistence of the potential climate impact on the economy, will translate into a lower value for $\lambda$, as will be seen.

Variants of the learning dynamics considered here are common in other fields of economics but some features of the setting deserve attention. Malueg and Tsutsui (1997) were among the first to consider learning of unknown Poisson rates in an R&D race; see also, for example, Keller, Rady, and Cripps (2005), and Bonatti and Hörner (2011). In this literature, new information is generated by periodic effort; no current effort means no new information. In our setting, one could also introduce effort for information acquisition so that news about impacts could arrive separately from experiencing them. However, in climate change it seems less natural to assume that pure research could produce robust information about how the physical reality interacts with the economy, without actual experienced impacts. The basic model introduced here connects the experience and learning in a stark way. The delayed learning follows from the arrival delay for impacts; even when losses are known to arrive for sure ($\mu_0 = 1$), they are expected to arrive with rate $\lambda$. This is clearly an approximation that calls for more structure; to this end, in Section 4, we allow the experience to generate information on a continual basis and, in Section 3.4, the arrival of new information depends on the temperature level. The latter extension connects to the literature on catastrophic environmental events where hazard rates for the high-consequence events are assumed to depend on past actions through their dependence on variables such as pollution stocks (Clarke and Reed 1994; Tsur and Zemel 1996; and, for example, Polaski, de Zeeuw, and Wagener 2011).

### 3.2 After learning, $I = 1$

To obtain the carbon price, that is, the social cost of current carbon emissions $z_t$, consider the effect of emissions at $t$ on a stream of future utilities. The full loss of utils per increase of temperatures as measured by $D_{t+\tau}$, caused by $z_t$ at time $t$, when discounted to $t$ with the model can be extended to include temperature brackets where impacts arrive almost surely, it is also reasonable to think that, for example, a long period of 2-degrees warming without impacts is evidence for not having impacts at such temperatures. Even if one considers “no news is good news” learning to be biased, this bias is consistent with the idea of having a conservative test against the climate policy ramp, as explained in the Introduction.
factor \(0 < \delta < 1\), is denoted by \(h\). It follows with the aid of (10) and (8):

\[
h \equiv - \sum_{\tau=1}^{\infty} \delta^\tau \frac{d\tau}{dz_t} = \Delta \sum_{\tau=1}^{\infty} \delta^\tau \mathcal{R}(\tau)
\]

\[
= \Delta \sum_{i \in I} a_i \pi \varepsilon \sum_{\tau=1}^{\infty} \delta^\tau (1 - \eta_i)^\tau - \delta^\tau (1 - \varepsilon_j)^\tau
\]

\[
= \delta \Delta \pi \frac{\varepsilon}{1 - \delta (1 - \varepsilon)} \sum_{i \in I} a_i \frac{1}{1 - \delta (1 - \eta_i)}.
\]

(12)

The present-value utility costs of current emissions can thus be compressed to a number, \(h\), that will be an input to the determination of the currently optimal carbon price. The first term, \(\delta \Delta \pi\), describes the utility loss associated with one emission unit when steady state damages would happen immediately at the next period. The second term discounts damages because of the time-delay associated with temperature adjustment. The third term with the summation describes the persistence of damages as the atmospheric \(CO_2\) stock decays slowly.

Remark 3 Conditional on \(I_t = 1\), the optimal carbon price is

\[
\tau_t = \frac{\partial y_t}{\partial z_t} = (1 - g) y_t \delta \Delta \pi \frac{\varepsilon}{1 - \delta (1 - \varepsilon)} \sum_{i \in I} a_i \frac{1}{1 - \delta (1 - \eta_i)}.
\]

(13)

Thus, the optimal carbon price in (14) is proportional to income, with proportionality depending only on \(\delta\), \(\Delta\), and the carbon cycle parameters in (8). Given loss parameter \(\Delta\), the same tax is optimal for any division between utility and production losses satisfying (10).

For the proof, given the Brock-Mirman structure (1972), the payoff implications of temperature changes are separable from capital wealth. The climate policy can be found by balancing the present-value of future utility costs of emissions (13) with the current utility-weighted marginal product of carbon:

\[
\frac{\partial u_t}{\partial c_t} = h. \quad \text{Since } \frac{\partial u_t}{\partial c_t} = 1/c_t = 1/(1 - g) y_t,
\]

we can express the optimal carbon price as

\[
\tau_t = \frac{\partial y_t}{\partial z_t} = h (1 - g) y_t
\]

(15)

which gives the result.

This, full information, optimal tax is proportional to income since, through the Brock-Mirman structure, we effectively assume a unit elasticity of losses with respect to income. This represents an intermediate position in the literature. Some economic climate-change losses, such as decreased agricultural yields in tropical areas, are likely to increase less
than one-to-one with income, as the share of the agricultural sector tends to decrease when income grows. At the same time, as these agricultural impacts are expected to be more severe in the currently warm-climate and less-developed countries, the share of damages in world-wide income will increase when those economies grow at rates larger than the world-wide average growth rate. Also, the monetary evaluation of economically intangible impacts such as ecological losses are expected to increase more than proportionally with income (Mendelsohn, Dinar and Williams 2006; Mendelsohn et al 2012).

3.3 Carbon price distribution before learning, $I = 0$

Once damages appear, the policies can be determined exactly as in Proposition 3. Prior to their appearance, the model generates a parametric distribution for the time when damages occur. Let $Z$ be the stochastic variable, measuring the full future utility cost from increasing current emissions $z_t$ marginally. Let $h_t = E_t Z$ be the expected present value of future utility losses associated with one unit of current emissions. $Z$ can take the values $Z_1, Z_2, \ldots$, where $Z_{\tau}$ is the current social cost of carbon if damages appear for the first time, precisely at period $t + \tau$. Thus, $Z_{\tau}$ characterizes the present-value marginal utility losses from current emissions $z_t$, assuming that the damage indicator $I_t$ remains at zero for all periods prior to $t + \tau$ but then turns positive. Proceeding as in Section 3.2 and using the emissions-temperature response from Section 2.2 we can obtain the present-value of such delayed utility losses in closed-form:

$$Z_{\tau} = \Delta \sum_{s=\tau}^{\infty} \delta^s R(s) \delta \sum_{i \in I} \frac{\pi a_i \varepsilon}{\varepsilon - \eta_i} \left( \frac{(1 - \eta_i)^{\tau}}{1 - \delta(1 - \eta_i)} - \frac{(1 - \varepsilon)^{\tau}}{1 - \delta(1 - \varepsilon)} \right).$$

Given our model of learning, we find for the distribution of $Z$ that

$$\Pr(Z = Z_{\tau}|I_t = 0) = \Pr(I_{\tau} = 1 \cap I_{\tau-1} = 0|I_t = 0)$$

which gives the probability that damages turn positive exactly after $\tau$ periods when the current time $t$ subjective belief for the climate problem is $\mu_t$. To find the corresponding cumulative distribution function for the utility losses, denoted by $F_t(Z)$, we first establish the probability that the damage has revealed itself at period $t$, irrespective of if $t$ is the
first time:

\[
\Pr(I_t = 1) = (1 - \mu_0) \Pr(I_t = 1|p = 0) + \mu_0 \Pr(I_t = 1|p = \lambda) = \\
\mu_0[1 - \Pr(I_t = 0|p = \lambda)] = \\
\mu_0[1 - \Pr(I_1 = \ldots = I_t = 0|p = \lambda)] = \\
\mu_0[1 - (1 - \lambda)^t].
\]

We can generalize this to expectations at period \(t\),

\[
\Pr(I_{t+\tau} = 1|I_t = 0) = \mu_t[1 - (1 - \lambda)^\tau]
\]

so that the distribution for \(Z\) is then given by

\[
F_t(Z_\tau) = \Pr(Z \leq Z_\tau|I_t = 0) = \Pr(I_{t+\tau-1} = 0|I_t = 0) = 1 - \mu_t + \mu_t(1 - \lambda)^{\tau-1}.
\]

We can use this distribution to determine the social cost of carbon at time \(t\) as dependent on beliefs \(\mu_t\).

**Theorem 1** Conditional on no experience of impacts by time \(t\) \((I_t = 0)\), the previous-period distribution of the social cost of carbon \(F_{t-1}(Z)\) stochastically dominates the current distribution \(F_t(Z)\). The social cost of carbon as measured by \(h_t = \mathbb{E}_t Z\) declines over time conditional on \(I_t = 0\). Moreover,

\[
h_t \equiv \mathbb{E}_t Z = \sum_{\tau=1}^{\infty} \delta^\tau \mathbb{E}_t \frac{du_{t+\tau}}{dz_t} = \mu_t h^l
\]

\[
h^l \equiv \delta \Delta \pi \sum_{\tau=1}^{\infty} \delta^\tau \sum_{i \in I} a_i (1 - \delta(1 - \eta_i)) (1 - \delta(1 - \lambda)) (1 - \delta(1 - \nu_i)) (1 - \delta(1 - \varepsilon)) (1 - \delta(1 - \lambda)).
\]

**Proof.** The expected utility losses from current emissions are equal to

\[
h_t = \mathbb{E}_t \Delta \sum_{\tau=1}^{\infty} \delta^\tau I_{t+\tau} \frac{dD_{t+\tau}}{dz_t} = \\
\Delta \sum_{\tau=1}^{\infty} \delta^\tau \Pr(I_{t+\tau} = 1|I_t = 0) R(\tau) = \\
\mu_t \Delta [\sum_{\tau=1}^{\infty} \delta^\tau R(\tau) - \sum_{\tau=1}^{\infty} (1 - \lambda)^\tau \delta^\tau R(\tau)].
\]

Using our temperature-response function leads to the expression for \(h_t\). Decreasing carbon prices measured in utils and stochastic dominance follow from (16) and \(\mu_t\) decreasing over time. ■
The result gives a closed-form expression for the optimal carbon price policy depending both on the climate system parameters and on the current belief of the damage distribution characterized by \((\mu_t, \lambda, \Delta)\). The first term that defines \(h^l\) equals \(h\), the full information policy variable (defined in (13)). The second term subtracts the present value of damages that in expectations do not occur, substituting \(\delta(1 - \lambda)\) for the discount factor.

Recall that the optimal carbon price is the income-weighted future utility-cost of current actions, analogous to (14), giving:

**Proposition 1** The optimal learning-adjusted carbon price is

\[
\tau_t = \mu_t h^l (1 - g) y_t.
\]

The results follow from the same arguments as for the full information case, using Theorem 1 for the expected future utility-costs. The tax rule follows from the first principles: the current utility-weighted gain from increasing emissions \(\left(\frac{\partial y_t}{\partial z_t} u'_t\right)\) should be equated with the \textit{current perception} of the future marginal loss in utils \(\left(\mu_t h^l\right)\). The externality internalized by the tax is thus dependent on beliefs on what the future costs from climate change might be — since beliefs change over time so does the perceived externality. Equation (17), while simple, makes the current tax to depend on the full state of the economy through \(y_t\). Moreover, \(\tau_t\) looks only at the cost side of current emissions by giving the money-metric social cost per ton of emissions; the deepness of the cuts in emissions induced depends on further details of the energy sector (provided in Section 3.5.4).

The tax rule now defines our experience-sensitive “climate policy ramp”. The gradually tightening carbon-price policy over time, can follow even with increasing climate optimism over time: despite the declining \(\mu_t\), sufficient growth of income \(y_t\) implies that the economy becomes, in expected terms, more exposed to losses from climate change. Limiting cases reveal the mechanisms at work. Consider time \(t = 0\), where the subjective belief of damages is given by \(\mu_0 < 1\). If damages are almost surely observable, \(\lambda \uparrow 1\), the optimal initial policy prior to experimentation is the full information policy, weighted with the subjective probability for damages, \(h^l \rightarrow h\). However, if damages do not appear the next period, \(I_1 = 0\), then the subjective assessment \(\mu_1\) drops to zero by the updating rule (11) as beliefs become very optimist, and the carbon price drops to zero, \(\mu_1 \searrow 0, h_1 = \mu_1 h^l \searrow 0\). In this case, no news reveals the true climate-economy state precisely. On the other hand, if climate change damages are not easily observable,
\( \lambda \downarrow 0 \), climate change is a problem with a non-significant rate of appearances in all cases and carbon prices are low, \( h^t \downarrow 0 \). But this case also implies that climate experiments are not very informative; there will be no learning, and the subjective assessment \( \mu_t \) in (11) remains almost unchanged over time.

The carbon price formula developed here differs from that in Golosov et al. (2014), who also build on the Brock-Mirman consumption choice framework for climate change impacts, in two main ways. First, our formula incorporates a delayed response of temperatures to atmospheric \( CO_2 \), without losing tractability. Golosov et al. assume that the temperature and associated potential impact of emissions reaches its maximum immediately after the date of emissions, which is hard to reconcile with the carbon cycle representations of the applied models typically used for carbon pricing. Second, we introduce a structure for beliefs and their tractable updating (11) so that the carbon price has a closed form and the contributions of beliefs and income become explicit in (17). Moreover, we will exploit in a following section the closed-form distribution of \( Z \) in (16) to connect the quantitative assessment to carbon price estimates in the literature.

### 3.4 Learning thresholds

Before moving to the quantitative assessment, it is useful to take a step towards the notion of temperature-dependent learning. In particular, for a conservative test against the climate policy ramp, we have assumed that any temperature increase generates information on the ultimate impacts of emissions; this allows optimism to work quickly against increasing carbon prices. The climate policy ramp follows by assumption if there are sufficient delays in learning — yet, it is useful state if the carbon price should grow faster than the economy. Suppose now learning takes place only above a temperature threshold, \( D_t \geq \overline{D} \), corresponding, for example, to 1 or 2 degrees Celsius above the pre-industrial temperature levels.

**Proposition 2** Assume that temperatures generate information on impacts only if \( D_t \geq \overline{D} \). Let \( D_0 < \overline{D} \) and \( t' < \infty \) be the first period such that \( D_t \geq \overline{D} \). Then, prior to \( t' \), the expected present-value utility impact of emissions increases over time: \( h_t < h_{t+1} \) for \( 0 < t < t' \).

\(^{17}\)Gerlagh and Liski (2013) compare the emissions-damage responses of DICE-2007, Golosov et al. 2014, and the one presented here. Moreover, the supplementary material of that paper contains a note that illustrates the importance of the non-monotonicity of the response in replicating the carbon price predictions of the applied climate-economy models.
Proof. Let $T$ be the set of periods $\tau$ such that $D_{\tau} \geq D$. The expected utility losses for $0 < t < t'$ satisfy

$$h_t = E_t \Delta \sum_{s=1}^{\infty} \delta^s I_{t+s} \frac{dD_{t+s}}{dz_t}$$

$$= \Delta \sum_{\tau \in T} \left\{ \Pr(I_{\tau} = 1 \cap I_{\tau-1} = 0|I_t = 0) \sum_{s=\tau-t}^{\infty} \delta^s \delta^s \frac{dD_{t+s}}{dz_t} \right\}$$

$$= \Delta \sum_{\tau \in T} \left\{ \Pr(I_{\tau} = 1 \cap I_{\tau-1} = 0|I_t = 0) \sum_{s=\tau-t}^{\infty} \delta^s R(s) \right\}$$

$$< \Delta \sum_{\tau \in T} \left\{ \Pr(I_{\tau} = 1 \cap I_{\tau-1} = 0|I_{t+1} = 0) \sum_{s=\tau-(t+1)}^{\infty} \delta^s R(s) \right\}$$

$$= \Delta \sum_{\tau \in T} \left\{ \Pr(I_{\tau} = 1 \cap I_{\tau-1} = 0|I_{t+1} = 0) \sum_{s=\tau-(t+1)}^{\infty} \delta^s \frac{dD_{t+1+s}}{dz_{t+1}} \right\}$$

$$= E_{t+1} \Delta \sum_{s=1}^{\infty} \delta^s I_{t+1+s} \frac{dD_{t+1+s}}{dz_{t+1}}$$

$$= h_{t+1}$$

The second line follows because $I_t = 0$ with certainty for $0 < t < t'$. The inequality follows as we subtract one period to take one period of delay away. The fifth line follows as beliefs do not change between $t$ and $t + 1$. ■

As long as no information can be obtained, no damages will occur but policy $h_t$ becomes more strict over time as the expected first appearance of damages comes closer. The tightening of policies continues until the temperatures start generating information.

**Proposition 3** For $0 < t < t'$, defined in Proposition 2, the optimal carbon tax grows faster than the economy.

Since the actual carbon tax is a multiple of income, the tax implied by $h_t$ for $D_t < D$ will be growing over time at a rate exceeding the growth of the economy, by Proposition 2. Further, recall that our emissions-temperature response implies that the temperature peak for a given emissions impulse lags 60-70 years behind the date of emissions: the learning of effects described here can start several decades after the emissions that caused climate change to break through the threshold. Thus, the shape of the emissions-temperature response, $R(\tau)$ is thus not only important as a measure of the development over time for the potential shock on the economy; it also dictates how quickly the climate experiment can become informative. This seems natural as the instrumental value of current emissions to generate sharper estimates before 2050 on the possibility of severe climate change impacts is, by the nature of climate change, very limited. But, in our
design of a conservative test for the climate policy ramp, we assume that any level of
temperature increase allows learning of the climate impacts on the economy in the quan-
titative analysis below. Here, we ignore the possibility of learning the climate impacts
from the shorter-term temperature volatility (see, for example, Kelly and Tan 2013); in
Section 4, we extend the tractable carbon price formula to this case.

3.5 Quantitative assessment

Throughout the quantitative analysis, we assume 10-year periods; the first year is ’2010’
corresponding to period 2006-2015. We assume only (potential) output losses from cli-
mate change so that $\Delta y = \Delta$ and $\Delta u = 0$, to maintain an easy comparison with earlier
studies. We take the Gross Global Product as 600 Trillion Euro [Teuro] for the decade,
2006-2015 (World Bank, using PPP). Throughout we assume a capital share of $\alpha = .3$
and one per cent pure rate of annual time preference, implying $\delta = .90$ for decadal periods
and resulting in savings $g = .27$. To focus on the gist of the quantitative conclusions, we
postpone the details of the energy sector, captured by the carbon-input dependent total
productivity $A_t(z_t)$ in the production function, to Section 3.5.4.

Normalizing the output loss parameter at unity, $\Delta = 1$, and assuming that these losses
exist at the outset gives us results comparable to Nordhaus’ (2007) baseline. Together
with our carbon cycle, such damages result in a carbon price of 22 EUR/tCO$_2$, equivalent
to about 105 USD/tC, for 2010. For an idea of the quantitative magnitudes, putting a
price of 22 EUR/tCO$_2$ on current annual global emissions represents about 1% of the
value of the world output. This estimate for the carbon price is higher than the Nordhaus
baseline (2007) because of our lower pure rate of time preference that facilitates the
calibration presented in Section 3.5.1.

3.5.1 Matching carbon price distributions

For an informed approach to quantifying the belief component in the model, we use now
a distribution of existing carbon price estimates as external data. The underlying idea in
this, admittedly unorthodox, calibration is that each number in the data presents a point

\footnote{Note that 1 tCO$_2$ = 3.67 tC, and 1 Euro is about 1.3 USD. Our number 105 USD/tC is almost
precisely equal to the DICE-2007 carbon price when in that model the elasticity of substitution parameter
is set to one and the pure rate of time preference is set to 1 per cent per year, as in our analytical model.
The number appearing in Nordhaus (2007), that is 35USD/tC, can be matched by setting 2.7 per cent
pure rate of time preference. However, Tol’s data, used in the calibration below, does not exist for this
value of time preference.}
estimate of the social cost. Our model gives a structural interpretation for the dispersion of the estimates, allowing calibration of the parameters that quantify the initial beliefs in the model. The policy-maker, that is, the decision-maker in the model, then forms one initial point estimate for the social cost, and evaluates its evolution over time given the learning dynamics assumed.\footnote{The social cost of carbon is an elusive concept in the applied work that has generated the data discussed below. Many of the studies do not optimize to find the optimal shadow value of the current carbon constraint; rather, the cost of carbon is the evaluated cost from a marginal increase of emissions given a background scenario for the economy; see, for example, the model descriptions in the Stanford Energy Modeling Forum (in Weyant et al, 2006). Our planner optimizes the social cost which, obviously, differs from the non-optimized estimates but is not necessarily inconsistent with them.}

Tol (2009) conducted a comprehensive survey of the existing estimates for the social cost of carbon. From a sample of 232 estimates he derived a distribution for the carbon price measured in 1995 USD/tC, controlling for the time discount rates used in the studies.\footnote{Tol (2014) corrects and updates for the original study; the reported modifications are inconsequential for our calibration, and thus the original data will be used here.} We focus on Tol’s sample corresponding to 1 percent pure rate of time preference.\footnote{Tol reports distributions for 0, 1 and 3 per cent discount rates, respectively. Our analysis of the 3-percent case produced very similar qualitative results; the levels of the policy variables are systematically lower.} Tol’s mean value for the carbon price is 32.7 for 2010 EUR/tCO$_2$ (his Table 2, 2009). We calibrate the climate system parameters as reported in Section 2.2 and choose economic parameters as stated above, and then fit our cumulative damage distribution function $F(Z)$ by choosing the initial prior $\mu_0$, hazard rate $\lambda$, and damage parameter $\Delta$. Note that in this interpretation of the data, the heterogeneity in the point estimates comes from different possible outcomes for the arrival date of the damage.

Fig. 2 depicts a spline connecting the 33, 50, 67, 90 and 95 percentiles of the carbon price distribution, expressed in 2010 EUR/tCO$_2$, as reported by Tol, jointly with the distribution that follows from our calibration, depicted as a smooth line. We can match the two cumulative distributions either by minimizing the errors at the reported percentile points, or, more directly, by matching the means and the end-points of the distributions. The approaches are almost outcome-equivalent. We followed the latter approach to allow for the interpretation set out below.

There is a mass point at zero, corresponding to a 20 per cent assessment that insignificant or positive climate change impacts will occur.\footnote{This number we inferred from Tol (2008).} For interpretation, we may think that $1 - \mu_0$ represents the share of climate experts having the assessment that
climate-change impacts will be negligible or even positive; to match the lower end of the distribution, we set $\mu_0 = .8$.

In the other extreme, there are experts who have strong views that income losses are high, arrive almost surely and soon: the high end of the carbon prices pins down the value for $\Delta_y$, assuming immediate sure loss of output. To avoid giving too much weight to a few extreme cost estimates in the sample, we truncated the fitted distribution at 87.7 EUR/tCO$_2$ by setting $\Delta_y = 4$. That is, maximum damages are by factor four higher than the middle-of-the-road damages assumed in Nordhaus (2007) — the implied output loss is then about 10.7 per cent from doubling the CO$_2$ stock.

The continuum of views between the extremes are described through the third parameter, $\lambda$. We obtain the value $\lambda = .077$ such that the initial carbon price implied by our model exactly matches Tol’s mean value of 32.7. Choice $\lambda = .077$ means that information is generated very slowly – there is about 8 per cent probability of learning per decade. A geometric distribution with this arrival rate per decade means that the expected arrival time for a severe climate change damage event is about 130 years. After 100 years without damages, the posterior for the eventual impact arrival $\mu_t$ is still 64 per cent.

This interpretation of Tol’s distribution creates a structural relationship between the calibrated $\lambda$ and the model description of the climate system. In particular, our climate model captures the persistence of the potential impacts from global warming through the carbon cycle stocks that have differing decay rates for the carbon contained in them. The greater is the share of low-decay carbon stock, the larger is the persistence. Changing the persistence of climate change will also change the calibrated $\lambda = .077$, to produce the
same match with Tol’s distribution. For illustration, increasing the share of zero-decay carbon, simultaneously decreasing the share of the fast-decaying carbon, from 16 per cent to 17, 18, and 21 per cent, leads to $\lambda$ equalling .07, .06, .05, respectively. Moving to the other direction, to less persistent impacts, has almost symmetrical implications; the learning rate becomes larger. Thus, through our structural interpretation, the calibration links the persistences of damages to the rate at which one can rule out severe damages from occurring. Moreover, after varying the persistence, we tend to conclude that the ballpark estimate for the impact arrival rate is relatively stable if one accepts the model structure with the decaying carbon stocks; one would need to assume a climate-system description that is far off the one in the literature to overturn the climate policy ramp described next.

### 3.5.2 The climate-policy ramp

The above calibration sets the optimal initial carbon price at the mean in Tol’s survey: 32.7 EUR/tCO$_2$ in 2010. Consider now the development of the optimal carbon price over time. We set $D = 0$, assuming that any level of temperature increase produces information.

Given the closed-form formula for the carbon price in (17), one approach is to conjecture future output or income levels, say, in 2050 and, conditional on no observed impacts by that time, obtain the future carbon price for that state of the world. However, future states of the world result partly from past policy decisions; carbon pricing decisions shape the current, and through investments, also future income levels. For consistent policy scenarios, in Section 3.5.4 we specify and calibrate a structure for the energy sector and total productivities through $A_t(z_t)$ in the production function. The scenarios presented here are based on this specification.

The benchmark for our assessment is the “Climate policy ramp” (dotted line in Fig. 3), based on Nordhaus’ DICE (2007) middle-of-the-road damage estimate, corresponding to $\Delta_y = 1$ sure-loss damages; that is, damages are immediately observed with no uncertainty. For 2010, with 1 per cent annual pure time-discounting and log-utility, the benchmark sure-loss policy path gives 22 EUR/tCO$_2$ as the optimal price which is almost identical to what DICE produces under this choice for discounting and preferences.

---

23We demonstrate this in detail in the online Appendix. First, in Fig. 6 we show that the DICE utility function can be changed to log-utility, while adjusting suitably the discount rate, without much affecting the baseline climate policy ramp. Second, using 1 per cent annual pure time-discounting (as in Tol’s
the coming century, typical for most no-uncertainty climate-policy assessments.

We now look at the optimal time path for the carbon price for high potential damages, but conditional on not observing these damages; that is, we consider the evolution of the policy when future impacts are potentially severe, $\Delta y = 4$, as determined by the calibration procedure above, but when no news on climate impacts arrive. Then, we compare this policy path to the baseline. Without impacts, the economy is unaffected by climate change but, since the carbon policies are in place, emissions and output will be reduced below the business-as-usual path. The optimal carbon price is depicted as a solid line in Fig. 3 over the coming century and beyond. The two climate policies—one with immediate damages based on the central estimate, and the other with high but only potential damages and gradual updating of beliefs to the no-news situation—have the same shape for the first century. The main result of the quantitative assessment follows: policies should become tighter over time even if climate optimism increases. Strikingly, for this particular learning scenario, it takes close to 200 years without observed climate damages for beliefs to become optimistic enough for the carbon price to decline—the social cost of carbon declines very slowly.

To assess the shape of the carbon price path, we decompose its level into its two main components. Recall that the optimal carbon price is proportional to $h_t$ capturing the expected utility losses from current emissions, and to income $y_t$; $\tau_t = h_t(1 - g)y_t$; $h_t = \mu_t h^l$. See Table 1 for the contribution of income ($y_t$) and learning ($\mu_t$) to the carbon price. Expected income growth is prodigious; in our evaluation, based on the IPCC scenarios (see Section 3.5.4 below), income rises five-fold during the coming century. Such an estimate is not unheard of, and is driven by an increasing population and the rise of the middle class in emerging economies. The development of beliefs is captured through $\mu_t$ in the Table. Observing no major climate damages over the coming century, leads to substantial increase in optimism, but, as is evident from the Table, it is the changing sample) and log-utility, we then compare the DICE policy ramp to that produced by our model in Fig. 7. Essentially, the striking similarity of the two paths follows since our carbon cycle and temperature delay dynamics come close to those in DICE.

24 The difference in levels follows since the baseline estimate by Nordhaus is close to the median, but lower than the mean in Tol’s distribution.

25 It is illuminating to consider the units of measurement for the utility loss measure $h_t = \mu_t h^l$, which has the same unit as the constant in the legend of the table: years per emissions. The variable $h_t$ measures the life-time equivalent of welfare that is lost per unit of emissions. For the year 2010, annual emissions are about .04 TtonCO$_2$, implying $.75 \times .04 = .03$ years of expected life-time destroyed by these emissions.
Figure 3: The carbon price for a sure income loss of 2.7 per cent from doubling the carbon concentration ($\mu = 1; \Delta_y = 1$), and for uncertain damages, conditional on no news on damages ($\mu_0 = 0.8; \lambda = 0.077, \Delta_y = 4$).

scale of the global economy that matters for the development of carbon pricing. The stake affected by the potential inverse income shocks from climate change increase so much that stabilizing carbon prices at the initial level — thus ruling out a climate policy ramp completely — would require that the climate experiment is by orders of magnitude more informative than considered here. The assessment of the probability of major utility losses, as captured by $\mu_t$, would need to decline by 50 per cent by 2050. For the belief updating to achieve this, the calibrated $\lambda$ we would need to be by factor four larger; it would require a very different climate-system description for the calibration to produce such a value for $\lambda$.

But how can carbon prices continue to rise as emissions go down and the climate returns to its natural state? Consider the persistence of atmospheric carbon, as shown in Figure 1. Current atmospheric CO$_2$ concentrations are about 400 particles per million (ppm), 125 ppm above the pre-industrial levels of 275 ppm. Even when CO$_2$ emissions fall to zero before 2100, it is expected that atmospheric CO$_2$ concentrations will not drop below 400 ppm before the end of the century, and stay above 350 ppm for centuries to come. In that sense, the climate is not expected to return to its natural state for a long period. How is this persistence captured by the model? Only indirectly through the calibration. The calibrated arrival rate for information, $\lambda$, captures how persistent is the belief for the ultimate damages — to match the data for carbon prices, $\lambda$ must become
Table 1: Decomposing the contribution of income and learning to the carbon price. Multiplying the first column and the second column, with a constant $h^T(1 - g) = 0.68\ [yr/Tt\text{CO}_2]$, gives the last column.

smaller for more long-lived climate impacts, as we have demonstrated.

### 3.5.3 Bad news

The climate policy ramp, without observed damages, is a function of beliefs and the severity of potential outcomes. We have quantitatively demonstrated how the persistence of policies depends on the slow decline of the posterior for damages but not yet how the bad news arrival affects the economy. For the latter, we define next the bad news carbon price as the socially optimal price at time $t$ if bad news arrive at time $t$. That is, $I_t = 1$ holds for the first time at $t$ so that the income-dependent carbon price is the one determined in Section 3.2. Fig. [ ] depicts both the no news and bad news carbon price paths for the near and longer terms. Note that the bad news price path is “virtual” because it is drawn for an economy in which the output is not dynamically adjusted due to past damages to obtain the bad news price for any given $t$; otherwise, time $t$ would not be the first arrival date of damages.$^{26}$ The starting level is given by our calibration at 87.7 EUR/tCO$_2$, as this is the highest price estimate that we pulled from Tol’s survey and applied to the immediate arrival of impacts. The increase in the virtual price over time reflects purely the effect of income growth on carbon pricing: the marginal utility loss from current emissions reductions, $\frac{\partial y_t}{\partial z_t}u'_t$, weigh less and less over time while the future marginal utility loss of current emissions, $h$, remains the same.

26 The immediate loss of output at time $t$ is accounted for in the calculation of the tax but then again ignored when moving to $t + 1$ to obtain a consistent bad news price for $t + 1$. 

<table>
<thead>
<tr>
<th>income beliefs, $\mu_t$</th>
<th>carbon price</th>
</tr>
</thead>
<tbody>
<tr>
<td>[T€/yr]</td>
<td>[.]</td>
</tr>
<tr>
<td>2010</td>
<td>60</td>
</tr>
<tr>
<td>2050</td>
<td>146</td>
</tr>
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</tr>
<tr>
<td>2200</td>
<td>703</td>
</tr>
</tbody>
</table>
Figure 4: Bad news and the no news carbon price

3.5.4 Decarbonization

We come to the end of the quantitative assessment by looking into the specific structure for the economy’s production function \(5\), used for the quantitative conclusions above. The specification is set up for a transparent calibration of the total and energy sector productivities. We assume

\[
y_t = k_t^\alpha [A_t(l_{y,t}, e_t)]^{1-\alpha} \exp(-\Delta y_t D_t) \quad (18)
\]

\[
A_t(l_{y,t}, e_t) = \min \{A_y l_{y,t}, A_e e_t\} \quad (19)
\]

\[
e_t = e_{f,t} + e_{n,t} \quad (20)
\]

\[
e_{f,t} = \min \{A_{f,t} l_{f,t}, B_t z_t\} \quad (21)
\]

\[
e_{n,t} = \frac{\varphi + 1}{\varphi} (A_{n,t} l_{n,t})^{\frac{\varphi}{\varphi+1}} \quad (22)
\]

\[
l_t = l_{f,t} + l_{n,t} + l_{y,t} \quad (23)
\]

There are time-trends for total labor \(l_t\), and for labor productivities \((A_y, A_e, A_f, A_n)\) in output, total energy, fossil-fuel energy, and non-carbon energy production, respectively. Total energy, \(e_t\), depends effectively only on labor allocation at time \(t\): the core allocation problem in the energy sector is how to allocate a given total labor \(l_t\) at time \(t\) between final output \(l_{y,t}\), fossil-fuel energy, \(l_{f,t}\), and non-carbon energy, \(l_{n,t}\). Thus, the climate policy steers the labor allocation \((l_{y,t}, l_{f,t}, l_{n,t})_{t\geq 0}\) and thereby the quantities of fossil-fuel,
Both energy sources are intermediates, summing up to the total energy input, $e_t = e_{f,t} + e_{n,t}$. The allocation outcome depends only on time and carbon inputs; labor-energy composite $[A_t(l_{y,t}, e_t)]$ then defines the total factor productivity term $A_t(z_t)$ in the economy’s production function (5).

Labor-energy composite $A_t(l_{y,t}, e_t)$ takes a Leontief form capturing an extremely low elasticity of substitution between labor in the final-good sector $l_{y,t}$ and energy $e_t$. By this assumption, we avoid unrealistically deep early reductions of emissions through substitution of labor inputs, and thereby approximate the energy sector capital adjustment delays; see also Hassler, Krusell and Olovsson (2012). A cost of the assumption is that the output-energy intensity remains fixed so that energy savings cannot arise as a source of emissions reductions; the reduction path for emissions in Fig. 5 is achieved through decarbonization, that is, substitution away from carbon energy.

In (21), we assume that $e_{f,t}$ can be produced with a constant-returns to scale technology using labor $l_{f,t}$ and the fossil-fuel $z_t$, where $A_{f,t}$ and $B_t$ describe productivities. The fuel resource is not a fixed factor and commands no resource rent; as in Golosov et al. (2014), the fossil-fuel resource is in principle unlimited. In contrast, in equation (22), where $\varphi > 0$ describes the elasticity of supply from the non-carbon sector; the non-fossil fuel energy production is land-intensive and subject to diminishing returns and land rents (as in Fischer and Newell, 2008).

Without carbon policy, $\tau = 0$, the labor market allocation can be solved in closed form; thus, we can invert the model to map from quantities $(l, y, e_{f}, e_{n})_{t \geq 0}$ to productivities $(A_y, A_e, A_f, A_n)_{t \geq 0}$ (see Appendix for the solution, and the supplementary material for the quantitative values). To express all energy in carbon units, we set $B_t = 1$, leaving us three distinct energy productivities $(A_e, A_f, A_n)$. We match the business-as-usual (BAU) quantities $(y, e_{f}, e_{n})_{t \geq 0}$ with the A1F1 SRES scenario from the IPCC (2007). Population follows a logistic growth curve based on World Bank forecasts. Population in 2010 is set at 6.9 [billion], while the maximum population growth rate is chosen such that in 2010 the effective population growth rate per decade equals 0.12 [/decade]. The maximum expected population (reached at about 2200) is set at 11 [billion]. Using these calibrated productivity trends, we produce the adjustment path of the economy for the optimal policies in Figs 3-5.

In Fig. 5 we depict the development of the energy sector when the economy does not experience climate-change impacts but faces the optimal no news tax as shown in Fig. 3. All energy is measured in \textit{CO}_2-equivalents; “carbon energy” gives then directly the carbon dioxide emissions per decade. Interestingly, the economy becomes fully decarbonized even
without observed impacts. Clearly, the result depends on the substitution possibilities between carbon and non-carbon energy that we discuss in detail below. On reflection, the decarbonization without impacts is not unreasonable; the optimal carbon price levels imply substantial value providing incentives for the transition. Current $CO_2$ emissions exceed 30 Gigatons annually, while the annual world output is about 60 trillion euro. A worldwide carbon price of 100 $EUR/tCO_2$ then represents about 5% of the value of the output, giving a ballpark idea of the decarbonization incentives. Most climate-economy models, including Nordhaus DICE-2007, produce decarbonization at such price levels that we, in our model, reach by 2080 (Fig. 3), explaining the decarbonization in Fig. 5.

![Figure 5: Decarbonization without observed climate impacts. Energy per decade measured in Teratons of $CO_2$ equivalents.](image)

4 Gradual learning from experienced damages

How much should the social cost of carbon rise after particular observational outcomes such as “hot summer spells” or, potentially, decline after periods of improved climate conditions? Gradual information arrival is an important element for the process of learning in climate change, especially when considering the typical (more moderate) damages foreseen in IAMs. The extension, presented next, preserves the climate-economy struc-
ture introduced so far and allows developing an explicit carbon price distribution for relatively general uncertainties concerning both income and climate impacts – the long-run distribution shows that, also in this more general setting, income is expected to be the key driver of the expected social cost of carbon.

4.1 The setting

Keeping our “Brock-Mirman climate-economy model”, including the (deterministic) temperature dynamics described in Section 2.2, we now redefine how the climate impacts the economy and how these impacts generate information — the physical description of climate change remains unaltered but the temperature increases impact the economy every period, although the true effect is hampered by noise. For a parsimonious exposition, assume that all impacts enter the utility function directly:

\[ u_t = \ln(c_t) - \exp(\Lambda_t) D_t, \]

where the state of the climate captured by \( D_t \) is observed, and losses contain a stochastic signal for the persistent damage sensitivity

\[ \Lambda_t = \Delta + \varepsilon_t \]

with \( \varepsilon_t \sim N(0, \sigma^2_{\varepsilon}) \) and i.i.d. across periods. We ”experience” the climate and thus observe \( \Lambda_t \) but cannot tell apart the contributions of noise and true damage where the latter has an initial prior \( \Delta \sim N(\mu_\Delta, \sigma^2_\Delta) \) with \( \mu_\Delta > 0 \). Thus, corresponding to our basic model in section 2.1, here \( \{\Delta_{y,t}, \Delta_{u,t}\} = \{0, \exp(\Lambda_t)\} \) for all \( t \).

Then, for this information structure, we can apply the normal learning rule (De Groot, 1970) to obtain, after \( t \) observations, the posterior distribution for \( \Delta \), at time \( t \) given by the parameters \( \mu_\Delta(t) \) and \( \sigma^2_{\Delta}(t) \):

\[ \mu_\Delta(t) \equiv \mu_\Delta + \frac{t \sigma^2_\Delta}{t \sigma^2_\Delta + \sigma^2_{\varepsilon}} (\overline{\Lambda_t} - \mu_\Delta) \] \hspace{1cm} (24)

\[ \sigma^2_{\Delta}(t) \equiv \frac{\sigma^2_\Delta \sigma^2_{\varepsilon}}{t \sigma^2_\Delta + \sigma^2_{\varepsilon}} \] \hspace{1cm} (25)

where \( \overline{\Lambda_t} \) is the average observed \( \Lambda_t \) after \( t \) observations.

The current time \( t \) perception of utility losses is also the best estimate of future losses at some \( \tau > t \) so that \( \Lambda_\tau \sim N(\mu_\Delta(t), \sigma^2_{\Delta}(t) + \sigma^2_{\varepsilon}) \). But, the expected future utility loss is

\[ \text{27} \] Since, by Remark 2, the productivity impacts and direct utility losses have interchangeable utility implications, this assumption is inconsequential for the policy rules developed here, although their quantitative assessment obviously depends on if the production possibilities are affected by climate change.
log-normal; it has a right-skewed distribution with a relatively fat tail for large damages; the expected future utility loss at \( \tau > t \) from a marginal temperature increase is given by

\[
E_t[\exp(\Lambda_\tau)] = \exp(\mu_\Delta(t) + \frac{1}{2}\sigma_\Delta^2(t) + \frac{1}{2}\sigma_\varepsilon^2).
\]  

(26)

4.2 Optimal policy

The current optimal policy follows the same principle as before: the current utility-weighted gain from increasing emissions \( \frac{\partial y_t}{\partial z_t} u_t' \) should be equalized with the present-value expected future utility losses \( h_t \). In this extended model, the policy variable \( h_t \) is obtained with the aid of (26).

\[
h_t = \exp(\mu_\Delta(t) + \frac{1}{2}\sigma_\Delta^2(t) + \frac{1}{2}\sigma_\varepsilon^2) \delta \pi \frac{\varepsilon}{1 - \delta(1 - \varepsilon)} \sum_{i \in I} \frac{a_i}{1 - \delta(1 - \eta_i)}.
\]

The formula here resembles that in [13]; but, whereas in the discrete-damage model the planner knows the future utility loss (if it happens) precisely, as captured by the coefficient \( \Delta \), in the current model the planner corrects for the posterior uncertainty. Thus, \( \Delta \) is replaced by \( \exp(\mu_\Delta(t) + \frac{1}{2}\sigma_\Delta^2(t) + \frac{1}{2}\sigma_\varepsilon^2) \).

For the exposition, we define the sure-loss policy through a constant policy variable \( \hat{h} \), which gives the present-value utility losses for initial mean expectation \( \mu_\Delta \) and no uncertainty \( \sigma_\Delta^2 = 0 \); that is

\[
\hat{h} = \exp(\mu_\Delta + \frac{1}{2}\sigma_\varepsilon^2) \delta \pi \frac{\varepsilon}{1 - \delta(1 - \varepsilon)} \sum_{i \in I} \frac{a_i}{1 - \delta(1 - \eta_i)}.
\]

For full information, the planner still corrects for the idiosyncratic component, \( \frac{1}{2}\sigma_\varepsilon^2 \), that is known to remain in existence for all future periods. We can now state:

**Proposition 4** The optimal learning-adjusted carbon price is proportional to the sure-loss policy:

\[
\tau_t = h_t(1 - g)y_t
\]

(27)

where \( h_t = \exp(\mu_\Delta(t) + \frac{1}{2}\sigma_\Delta^2(t) - \mu_\Delta) \hat{h} \),

(28)

\[\text{If climate change generates losses of output, or if output is stochastic for other reasons, the future temperature increase is uncertain; this uncertainty also contributes to the experienced utility loss. However, this is inconsequential for the optimal policy rule described below, as the marginal utility loss is independent of } D_t.\]
with the initial carbon-price markup defined by

\[ h_0 / \bar{h} = \exp(\frac{1}{2} \sigma^2_\Delta). \] (29)

**Proof.** The first equation in the proposition follows immediately from the Brock-Mirman structure of the model and the interpretation of \( h_t \) as the carbon price in utils. The second and third equations follow straight from the definitions above. ■

Importantly, the policy at \( t = 0 \) immediately deviates from the sure-loss policy \( h \), as the optimal carbon price at \( t = 0 \) now demands a markup due to the possibility of high-damage future events. Obviously, the decision maker does not expect a systematic deviation from the current mean damage in the sense that \( \mathbb{E}_t \mu_\Delta(t) = \mu_\Delta(t) \) for \( \tau > t \). However, there is a systematic drift in uncertainty as \( \frac{1}{2} \sigma^2_\Delta(t) \) goes down with more observations and thus over time. The substantial implication is, then, that the initial mark-up in the carbon price, due to the potential fat-tail of damages, is gradually expected to decline over time. While formally the result is not the same as the ”no news is good news” in our main model with one-time resolution of uncertainty, it is similar in the sense that uncertainty is expected to decrease over time and that, all else equal, puts a downward pressure on the carbon price.

It is possible to entertain the idea, as for the main model, that the uncertainty arises from differing subjective views on the true social cost of carbon, held by the climate policy experts. The distribution and the decision model described here compresses those views into a point estimate that is updated over time according to the learning processed assumed. A rough calibration involves fitting a log-normal distribution to the actual distribution of the social cost estimates (as reviewed by Tol, 2009). This has been done in van den Bijgaart et al. (2013) who, through simulation methods, also decompose the contribution of various sources of uncertainties to the carbon price estimates. The uncertainties, excluding the discount rate, lead to a carbon price markup of close to 70 \%; see their table 1. Setting a standard deviation of \( \sigma_\Delta = 1 \) for the current model results in an initial markup equal to \( \exp(0.5) - 1 \); that is, 65 per cent. We will use this ballpark number shortly to illustrate the quantitative magnitudes.

### 4.3 Carbon price distribution

The carbon price depends on our assessment of future climate-change impacts but also on output, \( \tau_t = h_t(1 - g) y_t \). We provide now a sharp assessment on which uncertainty

\(^{29}\)The discount rate is left out since the data presents a distribution of the social cost for a given discount rate.
can be expected to dominate in future social cost determination. To this end, assume that the income process follows a geometric random walk with a drift $\gamma$ and variance $\sigma_\gamma^2$. Let $\gamma_t = \ln(y_t/y_0)/t$ be the loglinearized growth rate.

**Proposition 5** The outcome for the optimal carbon price at time $t \geq 0$ satisfies

$$\ln(\tau_t/\tau_0) = t \ln(\gamma_t) + \ln(h_t/h_0),$$

with the distribution of future taxes, from the perspective of time $t = 0$, given by

$$E_0[\ln(\tau_t/\tau_0)] = t\gamma - \frac{1}{2} \frac{t \sigma_\Delta^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} \sigma_\Delta^2,$$

$$Var_0[\ln(\tau_t/\tau_0)] = t \sigma_\gamma^2 + \sigma_\Delta^2.$$  

**Proof.** The first equation follows from (27) combined with the income drift. For $E_0[\ln(\tau_t/\tau_0)]$, the first term on the right has trend equal to the trend of income growth, $\gamma$. The second term on the right has a trend that is, through (28), given by $E_0[\ln(h_t/h_0)] = \frac{1}{2} [\sigma_\Delta^2(t) - \sigma_\Delta^2]$, which becomes (25):

$$1 - \frac{1}{2} \frac{\sigma_\varepsilon^2 \sigma_\varepsilon^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} - \frac{1}{2} \frac{t \sigma_\Delta^4 + \sigma_\varepsilon^2 \sigma_\Delta^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} = - \frac{1}{2} \frac{t \sigma_\Delta^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} \sigma_\varepsilon^2.$$

For the last equation of the proposition, the variation of $t \ln(\gamma_t)$ is $t \sigma_\gamma^2$, while the variation of $\ln(h_t/h_0)$ is, through (28), given by the variation of $\mu_\Delta(t)$ in (24):

$$Var_0[\mu_\Delta(t)] = \frac{t \sigma_\Delta^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} Var_0[\Lambda_t] = \frac{t \sigma_\Delta^2}{t \sigma_\Delta^2 + \sigma_\varepsilon^2} (\sigma_\Delta^2 + \frac{\sigma_\varepsilon^2}{t}) = \sigma_\Delta^2.$$

The first term on the right-hand side of (30) shows how the expected carbon price follows the drift in income, and the second term characterises the decrease in the uncertainty mark-up over time. In the long run, the income effect unambiguously dominates: $E_0[\ln(\tau_t/\tau_0)] \rightarrow t\gamma - \frac{1}{2} \sigma_\Delta^2$. The variance, however, also increases linearly in time. Dividing expectations by the standard deviation, we can derive the z-score as the measure for the probability that carbon prices have increased at time $t$, compared to time 0. We illustrate the mechanisms through some rough numbers. Consider ten-year periods and an average growth rate of 3 per cent per year, as in the post-WWII period, $\gamma = 0.3$, with variation in decadal growth of 10 per cent points, $\sigma_\gamma = 0.1$. Learning in climate change

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30The general productivity of the economy can be stochastic so that $A_t(z_t) = \tilde{A}_t \hat{A}_t(z_t)$ where $\tilde{A}_t$ follows a geometric Brownian process, and $\hat{A}_t(z_t)$ is the part depending on carbon inputs.
is very slow. Assume that the initial mark up in carbon prices due to uncertain damage estimates is 65 per cent, as above, so that $\sigma_\Delta = 1$. Furthermore, assume that the uncertainty mark up decreases by factor 2 over the next forty years, $\sigma_\Delta^2(t) = 0.5$, so that consistent with (25) for a 10-year period we set $\sigma_\Delta^2 = 4; \sigma_\Delta = 1$. The expected increase in the logarithm of the carbon price over one decade is then given by $E_0[\ln(\tau_1/\tau_0)] = 0.2$. After forty years, the expected increase in the (logarithm of the) carbon price is given by $E_0[\ln(\tau_4/\tau_0)] = 0.95$. The variation of the logarithm of the carbon price is given by 1.01 and 1.04, respectively, so that the probabilities for a carbon price rise are 57 per cent, and 82 per cent, respectively. That is, there is 43 and 18 per cent probability that some good news on limited climate damages combined with bad news on low economic growth warrant a reduction in the carbon price, over these periods.

5 Concluding Remarks

We developed a tractable climate-economy model that allows a stylized but transparent and self-contained quantitative assessment of the optimal carbon price when the impacts of climate change can only be learned gradually over time. Rather than producing another estimate for the carbon price, we took the distribution of the existing estimates as given and provided a structural interpretation for it, stemming from the strong subjective components in the estimates. This paper is the first attempt to use the current estimates in a quantitative assessment of the robustness of climate policies. The optimal carbon pricing policies are robust to significant delays in obtaining hard evidence on the socio-economic impacts of climate change; it is the size of the economy that drives the carbon price.

Since there is a pressing policy need for a meaningful estimate of the carbon price, it is important that the framework is detailed enough for replicating the more comprehensive applied climate-economy models that, despite their shortcomings, are currently used for regulatory purposes such as those reported in Greenstone et al. (2011). Other analytical approaches are often more stylized (e.g., Weitzman 2009); as such, they provide valuable insights but cannot directly contribute to the quantitative determination of the optimal policies. Our model, while still very stylized, has a tractable emissions-damage response building on the insights from the natural science literature that, when combined with the macro-economic approach of Golosov et al. (2014), enables us to construct a transparent quantitative policy tool, with an explicit component for beliefs.\footnote{The supplementary material of Gerlagh and Liski (2013) includes a note that evaluates numerically...}
For wider implications for future research, it could be interesting to explore how countries’ climate policies are shaped by own and other countries’ experiences of climate impacts through their effect on beliefs regarding the country-specific ultimate impacts. In general, it is well-received that past experiences affect current policy-decisions through beliefs. There are long traditions for beliefs and learning in the strand of literature on the design of optimal monetary policy; however, the issue of incorporating beliefs as determinants of the key macroeconomic choices together with a quantitative assessment has been recently developing (see, e.g. Buera 2011, et al.). This issue seems very relevant in the context of climate change: policymakers are domestically motivated but are certainly learning from experiences elsewhere.

References


[34] Maier-Reimer E. and K.Hasselman (1987), Transport and storage of CO2 in the ocean - an inorganic ocean-circulation carbon cycle model, Climate Dynamics 2, 63-90.


APPENDIX FOR ONLINE PUBLICATION
Appendix: solution to the detailed energy-sector model

The online supplementary file contains a program for reproducing the graphs in the text, https://www.dropbox.com/sh/7meos655j14jh5p/_dlr8X_FHI. The labor allocation is numerically obtained as follows. The allocation can be solved period-by-period taking the (i) productivity parameters, (ii) total labor, (iii) savings $g$, and (iv) carbon policies $h_t$ as given. We drop the time subscript in the variables:

1. We normalize prices for the final good to equalize marginal utility, so that factor prices can be interpreted as marginal welfare per factor endowment:

\[ p = \frac{1}{c} = \frac{1}{(1 - g)y}. \]

2. Final-good producers of $y$ take capital $k$, wages $w$, and prices of energy $q$ and output $p$ as given. Since $y = k^\alpha \left[ \min \{ A_y l_y, A_e e \} \right]^{1-\alpha} \exp(-\Delta_{y,t} D_t)$, factor compensation for labour and energy together receives a share $(1 - \alpha)$ of the value of output $py$:

\[ w l_y + q e = (1 - \alpha) p y \]

where $e = e_f + e_n$.

3. Fossil-fuel energy production combines labor and fuels, with technology $e_{f,t} = \min \{ A_{f,t} l_{f,t}, B_t z_t \}$. Fossil fuel use and labour employed, $z, l_f \geq 0$, are strictly positive if $q$ covers the factor payments, including the carbon price $\tau$

\[ \left[ q - \left( \frac{w}{A_f} + \frac{\tau}{B} \right) \right] \times l_f \leq 0. \]

The zero profit condition for fossil fuel energy allocates the value of fossil fuel energy to labour and emission payments; using the production identity we can express it in terms of labour employed,

\[ q e_f = w l_f + \tau z = (w + \frac{\tau A_f}{B}) l_f. \]

4. Carbon-free energy inverse supply is given by the first-order condition

\[ q = w \frac{\partial l_n}{\partial e_n} = \frac{w_n}{(A_n)^{\frac{1}{\tau+1}} (l_n)^{\frac{1}{\tau+1}}}. \]

The value share of labour employed in the carbon-free energy sector equals $\varphi/(1 + \varphi)$, so that the rent value is expressed in labour employed:

\[ q e_n = (1 + \frac{1}{\varphi}) w l_n \]
We obtain four equations in four unknowns $l_y,l_f,l_n, w$:

\begin{align}
A_y l_y &= A_e (A_f l_f + \frac{\varphi + 1}{\varphi} (A_n l_n)^{\frac{\varphi}{\varphi+1}}) \quad (32) \\
wl + \frac{\tau A_f}{B} l_f + \frac{1}{\varphi} w l_n &= \frac{1 - \alpha}{1 - g} \quad (33) \\
\frac{w}{A_f} + \frac{\tau}{B} &\geq \frac{w (l_n)^{\frac{1}{\varphi+1}}}{(A_n)^{\frac{1}{\varphi+1}}} \quad l_f \geq 0 \quad (34) \\
l_y + l_f + l_n &= l \quad (35)
\end{align}

For (32) note that, for strictly positive input prices, $A_t(\cdot) = \min \{A_y l_y, A_e e\} \Rightarrow A_y l_y = A_e e$. In equation (33) we allocate the value of output that is not attributed to capital (the right-hand side) to the labour, carbon emissions, and land rent for the non-carbon energy (where we latter two terms are expressed in labour units). Equation (34) compares the production costs for fossil fuel energy with non-carbon energy, and the last equation is the labor market clearing equation. Note that the solution depends on the state of the economy only through total labor $l$ and productivities $A_y, A_e, A_f, A_n$.

In the absence of a carbon policy, $\tau = 0$, we can solve the allocation in closed-form:

\begin{align}
l_{n,t} &= \frac{A_{n,t}^{\varphi}}{A_{f,t}^{\varphi+1}} \quad (36) \\
w_t &= \frac{1 - \alpha}{1 - g} \frac{\varphi}{\varphi l_t + l_{n,t}} \quad (37) \\
l_{y,t} &= \frac{A_{e,t}}{A_{y,t} + A_{e,t} A_{f,t}} [A_{f,t} (l_t - l_{n,t}) + \frac{\varphi + 1}{\varphi} (A_{n,t} l_{n,t})^{\frac{\varphi}{\varphi+1}}] \quad (38) \\
l_{f,t} &= l_t - l_{y,t} - l_{n,t} \quad (39)
\end{align}

Here we include the time subscripts to emphasize the drivers of the solution. This business-as-usual allocation is used to calibrate the productivities. When $\tau > 0$, the solution is numerical, and available in the supplementary file.
Appendix: DICE benchmark climate policy ramp

Figure 6: We consider the numbers presented in Nordhaus (2008), Table 5.4. These are the optimal carbon prices in the central DICE run. Then we take the DICE model, reproduce these numbers, with the elasticity of marginal utility=2. Then, we change the utility function to logarithmic and the discount rate using the Ramsey rule. This gives the DICE run with elasticity of marginal utility=1. The outcomes are almost identical; we can work with log-utility for the DICE benchmark that is considered in the next Figure.
Figure 7: This Figure continues from Figure 6. The DICE path depicted is obtained for log-utility (elasticity of marginal utility=1) and 1 % discount rate (which is lower discounting than implied by the DICE base run with marginal utility=2). Then, we use our model to produce the climate policy ramp, the GL path, using 1 % discount rate and the sure-loss damages corresponding to those in DICE ($\Delta y = 1$). This confirms that our sure-loss benchmark path is almost identical to what DICE produces under the same choices for discounting and preferences.