

Forward trading in exhaustible-resource oligopoly

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Abstract

We analyze oligopolistic exhaustible-resource depletion when firms can trade forward contracts on deliveries — a market structure prevalent in many resource commodity markets — and find that trading forwards can have substantial implications for resource depletion. We show that when the firms' initial resource stocks are equal, the subgame-perfect equilibrium path approaches the perfectly competitive (Hotelling) path as firms trade forwards frequently. But when firms have stocks of different sizes, they can credibly escape part of the competitive pressure of forward contracting. It is a unique feature of the resource model that equilibrium contracting and the degree of competition depends on resource endowments (JEL classification: G13, L13, Q30).

1 Introduction

Hotelling's (1931) theory of exhaustible-resource depletion is a building block for understanding intertemporal allocation of a finite resource stock. The theory is used in myriad

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of applications which, without exceptions known to us, assume implicitly or explicitly that the commodity stock is sold in the spot market only, thereby ruling out forward trading despite the fact that it is commonly observed in many commodity markets and markets for exhaustible-stocks in particular. Forward trading is typically associated to the desire of some groups of agents to hedge risks but it can also arise in oligopoly settings without uncertainty. As shown by Allaz and Vila (1993) for the case of reproducible commodities, the mere possibility of forward trading forces firms to compete both in the spot and forward markets, creating a prisoner's dilemma for firms in that they voluntarily sell forward contracts (i.e., take short positions in the forward market) and end up producing more than in the absence of the forward market. In this paper we are interested in understanding whether and how this pro-competitive effect of forward contracting can also arise in an oligopolistic exhaustible-resource market.¹

In exhaustible-resource markets firms face an intertemporal capacity constraint coming from their finite stocks. Hotelling (1931) establishes a simple principle for monopolistic allocation of the capacity over time: marginal value of using the capacity in different periods should be equalized in present value. Under standard assumptions, the resource depletion becomes more conservative when compared to the perfectly competitive path; the monopoly sales are shifted towards the future as a way to increase the value of early sales. An oligopoly follows the same (spot) allocation principle as the monopoly, with differences in outcome analogous to those that arise between static monopoly and oligopoly. Furthermore, this intertemporal capacity constraint rules out a direct application of Allaz and Vila (1993), because the overall output expansion is not possible. One may then conjecture that for exhaustible resources forward contracting leaves oligopoly rents intact (e.g., Lewis and Schmalensee, 1980; Ulph and Ulph, 1989).²

This conjecture is not correct, however. As in Allaz and Vila (1993), we find that forward contracting can introduce substantial competitive pressure, but, as we explain below, the mechanism delivering this pressure cannot be directly seen from their model. As in their model, our subgame-perfect equilibrium (SPE) strategies have a Markov structure in that they depend only on the current state of the market, which in our case corresponds to remaining stocks and existing forward positions. We show that

¹Phlips and Harstad (1990) already mentioned that forward contracting can have an important effect on oligopolistic exhaustible-resource markets but they did not explain whether and to what extent firms will sign forwards in equilibrium.

²Both Lewis and Schmalensee (1980) and Ulph and Ulph (1989) suggest that the existence of futures markets validates the use of "path strategies", or more generally, allows firms to commit to production plans.

when firms' initial stocks are of equal size the symmetric SPE results in a delivery path that converges to the perfectly competitive path as firms interactions become infinitely frequent, i.e., in the continuous-time limit. Qualitatively, this outcome is not different than in Allaz and Vila's (1993) when we let their (single) spot market be preceded by an infinitely large number of forward openings. However, when firms have resource stocks of different sizes, they can credibly contain part of the competitive pressure of forward contracting. It is a unique feature of the resource model that equilibrium contracting and the degree of competition depends on resource endowments.

To illustrate the logic of how forward contracting introduces competition in the symmetric case consider first a stock so small, or period length so large, that the one-period demand absorbs the stock without any storage. Forward contracting then plays no strategic role because the overall supply is in any case to be consumed in one period. Take now a shorter period length, or larger stock, so that consumption takes place over two periods. Contracting preceding spot sales now plays a role: it induces firms to race for a higher capacity share in the first period, the more profitable of the two periods. In effect, forward contracting moves supplies towards the present, leading to a more efficient allocation of the capacity. In the limit, when a given overall stock is sold arbitrarily frequently, firms have a large number of forward openings to race for the more profitable spot markets. The race ends when all spot markets are equally profitable, i.e., when the allocation is perfectly competitive, as in Hotelling (1931).³

The above logic changes when stocks are asymmetric. The smaller firm can now credibly use the forward market to increase its presence in the earlier (more profitable) markets because it knows the large firm will find it optimal to reallocate part of its stock to later markets in an effort to soften competition. In fact, in the two-period model, a sufficiently small firm can commit to fully exhaust in the first period by contracting its entire stock. If so, the larger firm has no contracting incentives. In this case, the small firm strictly benefits from the forward market in that it allows it to implement its most profitable, i.e., Stackelberg, outcome.⁴ Qualitatively, the equilibrium implies that the small firm free-rides on the large firm's market power and exits the market first, as in Salant (1976).⁵

³The use of forward contracting in the race for appropriating a larger share of the oligopoly rents resembles the use of private storage in the race for appropriating a larger share of a common and exhaustible resource in Gaudet et al. (2002).

⁴Note that adding more contracting opportunities before the spot openings does not change the outcome in this two period example.

⁵Salant (1976) considers a game in which a large supplier and a fringe of competitive suppliers choose

Our strategy of exposition is to start in Section 2 with a two-period model illustrating both of the above symmetric and asymmetric equilibria. While helpful in explaining the basic mechanism, the extensive form of the two-period model is incomplete because firms should be able to choose how long the market interaction lasts in equilibrium. For example, firm i may respond to firm j 's heavy contracting in period t by avoiding own contracting at t and allocating more capacity to a less contracted period $t + 1$ instead.⁶ Despite this limitation, the main conclusions of the two-period model carry through to a more general setting, although with considerable additional complexity.

In Section 3, we set up the general version of the model where deliveries and future contract positions are chosen on a period-by-period basis depending on current physical stocks and existing positions inherited from the past. In Section 3.2, we characterize the properties of the symmetric SPE. We also describe the contracting dynamics showing that contract positions are altered for all future dates in each forward market interaction. Then, we solve the continuous-time limit of the discrete model for the symmetric case and show how the equilibrium path converges to the perfectly competitive path.

We conclude this introductory section with a brief discussion of how this research relates to three strands of literature. First, our work is closely related to the basic exhaustible-resource theory under oligopolistic market structure. This literature has focused on developing less restrictive production strategies for firms (from "path" to "decision rule" strategies)⁷ and also on including more realistic extraction cost structure (towards stock-dependent costs).⁸ None of the papers in this literature explicitly consider the effect of forward trading on the equilibrium path. However, it is interesting that the resource-depletion path suggested by the two-period model is qualitatively similar to

simultaneously their entire production path at time zero. He shows that there will be two distinctive phases in equilibrium: a "competitive" phase with both type of players serving the market followed by a monopoly phase in which only the large supplier serves the market.

⁶This difference in extensive form is also an important difference relative to the Allaz and Vila (1993) model where firms are trapped to face the prisoners' dilemma in a particular spot market.

⁷Loury (1986), Polasky (1992), and Lewis and Schmalensee (1980) use path strategies; Salo and Tahvonen (2001), for example, use decision-rule strategies. See Reinganum and Stokey (1985) for discussion of this modeling choice, and illustration in a resource context. For a recent survey on the Hotelling model and its extensions, see Gaudet (2007).

⁸Salo and Tahvonen (2001) solve their model with stock-dependent costs, so that the overall amount of the resource used is endogenously determined in equilibrium. In this sense, the resource is only economically exhausted. In our model, the resource is physically exhausted as the cost of using it is independent of the stock level. We leave it open for future research how replacing physical capacity with economic capacity would alter the contracting incentives.

that in Salant (1976) where the overall sales period is also divided into two distinct phases. In Salant's model, there is a large supplier and fringe of competitive suppliers. All suppliers are active in the competitive phase, which is followed by a monopoly phase where only the large firm is active. Forward contracting among asymmetric firms leads to a qualitatively similar equilibrium pattern, although the mechanism is very different as well as the degree of competition arising from a given division of stocks.

Second, there is a recent literature on organization of trade in dynamic oligopolistic competition under capacity constraints (e.g., Dudey, 1992; Biglaiser and Vettas, 2008; Bhaskar, 2008; Talluri and Martinez, 2010). These papers focus on dynamic price competition and also on the efficiency losses and changes in division of surplus caused by strategic buyers. We depart from this literature by assuming non-strategic but forward looking buyers, and we consider quantity competition in two dimensions (spot and forward markets). Our result that the firm with smaller capacity sells first and at higher prices sounds similar to Dudey's (1992) but is, in fact, quite different. In our case the large firm is active throughout the equilibrium and makes larger profits overall; the small firm is only free-riding on the large firm's market power, much the same way the fringe is free-riding on the large firm's market power in Salant (1976).

Third, there is a literature on forward trading starting with Allaz and Vila (1993) who analyze a static Cournot market for a reproducible good. Mahenc and Salanie (2004) show that price competition can reverse the effect of forward trading on competition. Liski and Montero (2006), on the other hand, develop a repeated interaction model of forward and spot transactions showing that forward contracting can expand the scope for collusive behavior in part because the threat of falling into the non-cooperative outcome becomes more effective. Given the result of this paper that contracting can add substantial (non-cooperative) competitive pressure, it is obvious that the same pro-collusion argument applies in the resource context whenever the model is specified to allow for non-stationary strategies. There is also a recent empirical literature looking at the effect of forward contracting on the performance of some oligopoly markets, in particular, electricity markets (e.g., Wolak, 2000; Fabra and Toro, 2005; Bushnell *et al.*, 2008; Fabra and de Frutos, 2010).

2 Two-period model

The implications of forward contracting for the equilibrium of a depletable-stock oligopoly can be best explained by first considering a simple model with only two periods. This

section will also introduce the notation and assumptions that will be used throughout the paper.

2.1 Notation and assumptions

Consider two firms (i and j), each holding a stock of a perfectly storable homogenous good, denoted by s_1^i and s_1^j , respectively, to be sold in two periods ($t = 1, 2$). There are no production (or extraction) costs other than the shadow cost of not being able to sell tomorrow what is sold today. Firms discount future profits at the common discount factor $\delta < 1$.

Firms attend the spot market in both periods $t = 1, 2$ by simultaneously choosing quantities q_t^i and q_t^j . For tractability, we assume that the spot price at t , which is denoted by p_t^s , is given by the linear inverse demand function $p_t^s(q_t^i + q_t^j) = a - (q_t^i + q_t^j)$, where a is the (choke) price at which consumers (and firms) can buy a substitute good from a perfectly elastic supply.

Firms are also free to simultaneously buy or sell forward contracts that call for delivery of the good at any of the spot markets that follow. We treat these forward contracts as pure financial commitments, so rather than physically delivering or acquiring the good at the spot market firms can close their forward positions at the spot price $p^s \in [0, a]$.⁹

For each period we assume a two-stage structure: the forward market precedes the spot market where physical deliveries take place and contract positions are closed. In a forward market, firms can take positions for any future spot market, including the present period spot market (in this two period model no spot markets will open after $t = 2$). Forward contracts by firm i at $t = 1$ for the first and second spot markets are denoted by $f_{1,1}^i$ and $f_{1,2}^i$, respectively. Similarly, forward contracts at $t = 2$ for period-2 spot market is denoted by $f_{2,2}^i$. We adopt the convention that $f^i > 0$ when firm i is selling forward contracts (i.e., taking a short position) and $f^i < 0$ when is buying forwards (i.e., taking a long position). In this complete-information setting, we further assume that forward positions are observable and enforceable.¹⁰ Note that while position $f_{1,1}^i$ is an obligation that can be only closed at the spot market 1, the forward position for $t = 2$ can be

⁹This form of contracting is commonly known as a two-way contract for difference; see, for example, Green (1999). Note that like in Allaz and Vila (1993) and Mahenc and Salanie (2004) our results do not change if we restrict forward contracts to physical commitments and where firms could, if necessary, cover their short obligations with the substitute good.

¹⁰The assumptions for the contract market are the same as in Allaz and Vila (1993), Mahenc and Salanie (2005), and Liski and Montero (2006).

modified at the forward market in 2 by taking an additional (long or short) position $f_{2,2}^i$; thus, the aggregate position for period 2 would be equal to $F_2^i = f_{1,2}^i + f_{2,2}^i$. Finally, the forward clearing price (i.e., striking price) at t when taking a position for $\tau \geq t$ is denoted by $p_{t,\tau}^f$.

2.2 Pure-spot trading

To understand the effect of forward contracting on the equilibrium path it is useful to start with the case in which firms can only trade in the spot markets. A strategy for player i in this pure-spot game specifies an output for period 1, $q_1^i \leq s_1^i$, and an output for period 2, q_2^i , that is a function of the remaining stocks in period 2, s_2^i and s_2^j , where $s_2^i = s_1^i - q_1^i$. The SPE strategies are to be found by backward induction.

At $t = 2$ and given stocks $s_2^i \geq 0$ and $s_2^j \geq 0$, firm i 's best response to an output $q_2^j \geq 0$ by firm j is given by

$$R_2^i(q_2^j) = \min\{s_2^i, \arg \max_{q_2^i} p_2^s(q_2^i + q_2^j)q_2^i\} \quad (1)$$

which leads to the (unique) equilibrium strategy

$$q_2^i(s_2^i, s_2^j) = \begin{cases} a/3 & \text{if } s_2^i \geq a/3 \text{ and } s_2^j \geq a/3 \\ s_2^i & \text{if } s_2^i < a/3 \text{ and } s_2^j \geq 0 \\ R_2^i(s_2^j) & \text{if } s_2^i > a/3 \text{ and } s_2^j < a/3 \end{cases} \quad (2)$$

for both i and j . These equilibrium strategies are simply Cournot strategies with capacity constraints that result in output pairs within the inner envelope of the best-response functions.

Consider now decisions in the first period. Depending on the size of the initial stocks, s_1^i and s_1^j , we can have extreme cases in which firms never get to produce in period 2 or that their first period decisions do not affect what they produce in period 2. As shown in Figure 1, the former occurs when stocks lie in region (C,C), that is, when both firms are sufficiently capacity constrained by their initial stocks that they sell everything in period 1. The equilibrium condition for this to happen is that the marginal revenue (MR) of selling an extra unit in period 1 is equal to or greater than the marginal revenue of selling that unit in period 2 at price a , i.e., $MR_1^i \equiv a - 2s_1^i - s_1^j \geq \delta a$ for both i and j . The latter case, on the other hand, occurs when stocks are in the region (R,R), that is, when stocks are so large that firms behave as if they were selling a zero-cost reproducible good. The equilibrium condition for this is that marginal revenues equal zero in both periods and

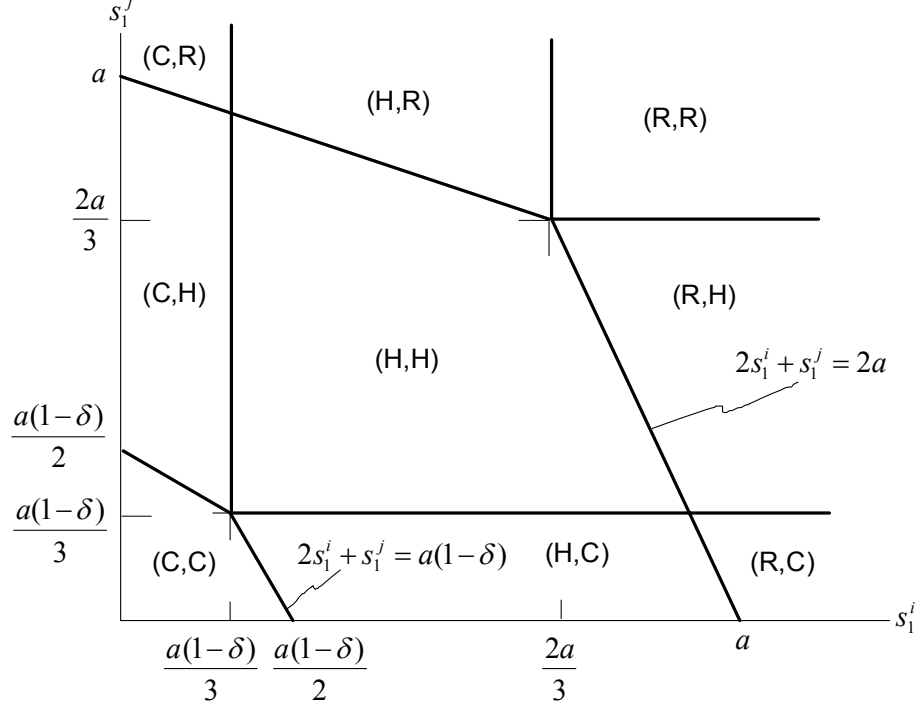


Figure 1: Spot trading equilibrium as a function of initial stocks

for both firms. It is clear that this happens when firms produce the Cournot output in each period, which requires $s_1^i, s_1^j \geq 2a/3$.

Figure 1 also depicts cases of initial stocks within these two extreme. When stocks are in region (H,C), for instance, we have that firm j is capacity constrained and sells everything in period 1 (i.e., $MR_1^j \geq \delta MR_2^j$) whereas firm i operates in a Hotelling world of scarcity rents, that is, according to Hotelling's equilibrium condition for an exhaustible resource (i.e., $\delta a \geq MR_1^i = \delta MR_2^i \geq 0$). Anticipating (2), for j to exhaust its stock in period 1 and i to exhaust his in period 2 requires, respectively

$$a - 2s_1^j - q_1^i \geq \delta(a - q_2^i) \quad (3)$$

$$\delta a \geq a - 2q_1^i - s_1^j = \delta(a - 2q_2^i) \geq 0 \quad (4)$$

where $q_2^i = s_1^i - q_1^i$. These equilibrium inequalities define the range of initial stocks that make region (H,C). It is obvious from (3) that j does not want to move any of its production to period 2. It is less obvious why could not we have an equilibrium in which j is forced to move part of its production to period 2 so both firms end-up producing in both periods. Such an equilibrium would require i to produce in period 1 beyond what is dictated by the equality in (4) resulting in $MR_1^i < \delta MR_2^i$; a violation of

the Hotelling equilibrium condition. Hence, when stocks are in region (H,C) the unique period-1 equilibrium strategies are given by $q_1^i = [a(1 - \delta) + 2\delta s_1^i - s_1^j]/2(1 + \delta)$ and $q_1^j = s_1^j$.

The remaining regions of Figure 1 are obtained in the same fashion. In region (H,H), both i and j 's production paths satisfy (4) with $q_1^i = [a(1 - \delta) + 3\delta s_1^i]/3(1 + \delta)$; in (R,H) firm i 's production path satisfies $MR_1^i = MR_2^i = 0$ while j 's satisfies (4) with $q_1^i = [2a(1 + 2\delta) - 3\delta s_1^j]/6(1 + \delta)$ and $q_1^j = [a(1 - \delta) + 3\delta s_1^j]/3(1 + \delta)$; and in (R,C) firm i 's path satisfies $MR_1^i = MR_2^i = 0$ while j 's satisfies (3) with $q_1^i = (a - s_1^j)/2$ and $q_1^j = s_1^j$.

Finally, note that from inequality (3) we can immediately see that $p_1^s > \delta p_2^s$. Unlike in a perfectly competitive exhaustible-resource market, here prices grow at a rate strictly lower than the interest rate (marginal revenues grow at the interest rate). In other words, oligopoly pricing depart from competitive pricing by shifting production from the present to the future.

2.3 Forward trading for symmetric stocks

We now allow firms to engage in forward trading in addition to spot trading. To facilitate the exposition we consider first the case in which firms have stocks of equal size $s_1^i = s_1^j > 0$, and leave the asymmetric case for the next section. A strategy for player i in this richer game specifies (i) a vector of forward quantities or positions for period 1, $\mathbf{f}_1^i = (f_{1,1}^i, f_{1,2}^i)$; (ii) an output for period 1 as a function of \mathbf{f}_1^i and \mathbf{f}_1^j ; (iii) a forward position for period 2, $f_{2,2}^i$, as a function of \mathbf{f}_1^i , \mathbf{f}_1^j and remaining stocks s_2^i and s_2^j , where $s_2^i = s_1^i - q_1^i \geq 0$; and (iv) an output for period 2, q_2^i , as a function of $F_2^i = f_{1,2}^i + f_{2,2}^i$, F_2^j and the remaining stocks s_2^i and s_2^j . Note that F_2^i represents firm i 's overall position for period 2.

As in the pure-spot game, the SPE is highly dependent on firms' initial stocks. If stocks are very small, for example, we will see that forward contracting plays no strategic role since everything is produced in the first period. If, on the other hand, stocks are extremely large, we are back to Allaz and Vila's (1993) analysis of forward contracting for reproducible goods (and where a fraction of the stock remains on the ground). Note that this result is particular to the two-period model because we are (artificially) restricting the number of spot market openings. In the general model the number of periods in which firms serve the spot market is endogenously determined, so no stock remains in the ground.

To show how the equilibrium transits from one extreme to the other, it helps to start by considering initial stocks large enough that firms serve both spot markets in

equilibrium and that firms are only allowed to sell/buy forwards for the first spot market, i.e., $f_{1,2}^i = f_{2,2}^i = 0$ for both i and j (we will see shortly that in some relevant cases this restriction in the number of forward openings is innocuous for the equilibrium). Working backwards and given stocks $s_2^i > 0$ and $s_2^j > 0$, firm i 's best response to an output $q_2^j \geq 0$ by firm j is given by (1). Invoking symmetry (i.e., $s_2^i = s_2^j$),¹¹ period-2 equilibrium quantities reduce to

$$q_2^i(s_2^i, s_2^j) = \min \{s_2^i, a/3\} \quad (5)$$

which is the standard Cournot outcome with capacity constraints.

Consider now decisions in the first spot market. Given $f_{1,1}^i$ and $f_{1,1}^j$, firm i 's best response to output q_1^j is given by

$$R_1^i(q_1^j, f_{1,1}^i, f_{1,1}^j) = \arg \max_{q_1^i} [W_1^i = p_1^s(\cdot)q_1^i + (p_{1,1}^f - p_1^s(\cdot))f_{1,1}^i + \delta p_2^s(\cdot)q_2^i(s_2^i, s_2^j)]$$

where W_1^i is firm i 's payoff at the spot market in period 1 and $q_2^i(s_2^i, s_2^j)$ is given by (5). Term $(p_{1,1}^f - p_1^s(\cdot))f_{1,1}^i$ is the open contract position that is to be closed immediately after the spot price is realized. Due to complete information, the equilibrium closing value of this position is zero but, nevertheless, firm i has an incentive to influence the value of the position in the spot market as the spot price is yet to be determined. We rearrange the payoff as

$$W_1^i = p_1^s(\cdot)(q_1^i - f_{1,1}^i) + \delta p_2^s(\cdot)q_2^i(\cdot) + p_{1,1}^f f_{1,1}^i \quad (6)$$

where $s_2^i = s_1^i - q_1^i$. Since the last term in (6) enters as a constant,¹² the relevant spot sale for firm i 's profits is not total production q_1^i but $q_1^i - f_{1,1}^i$. Firm i 's best response to q_1^j satisfies the intertemporal optimization principle that discounted marginal revenues should be equalized across periods, that is,

$$a - 2q_1^i - q_1^j + f_{1,1}^i = \delta(a - 2q_2^i - q_2^j) \geq 0 \quad (7)$$

where $q_2^i \leq s_1^i - q_1^i$.

Solving (7) for i and j , we obtain the period-1 equilibrium output

$$q_1^i(f_{1,1}^i, f_{1,1}^j) = \min \left\{ \frac{a + 2f_{1,1}^i - f_{1,1}^j}{3}, \frac{a(1 - \delta) + 3\delta s_1^i + 2f_{1,1}^i - f_{1,1}^j}{3(1 + \delta)} \right\} \geq 0 \quad (8)$$

¹¹Throughout this section and to save space, we will only focus on symmetric stocks and positions because it is the relevant case for computing the equilibrium path.

¹²Note that if forward contracts were defined as physical obligations, the term $p_{1,1}^f f_{1,1}^i$ would not appear in (6) because it constitutes a revenue the firm pockets at the forward stage. Under financial obligations, forward positions are closed at the spot stage, which explains why $p_{1,1}^f f_{1,1}^i$ enters in (6). Nevertheless, it enters as a constant, so it makes no difference for firms' spot decisions, and thereby, for their (strategic) forward decisions.

The first term in the bracket is the relevant equilibrium quantity when, in equilibrium, $q_1^i + q_2^i < s_1^i$ (i.e., $MR_1^i = \delta MR_2^i = 0$). This is the reproducible-good case where the effect of forward contracting is well documented in Allaz and Vila (1993). Whenever firms hold short positions, $f_{1,1}^i, f_{1,1}^j > 0$, they care less about the price effect of an increase in production and therefore end up producing above the Cournot total output.¹³

The more relevant case for our analysis, however, is when in equilibrium $q_1^i + q_2^i = s_1^i$ for both i and j (i.e., $MR_1^i = \delta MR_2^i > 0$). When the stock is fully exhausted, the period-1 equilibrium output is given by the second term in (8). Despite a firm's total production is limited by its stock, the second term in (8) shows that forward positions can still have an effect on spot competition in this exhaustible-resource setting by moving production across periods (if firms hold no forward positions, i.e., $f_{1,1}^i = f_{1,1}^j = 0$, we obtain the oligopoly solution for region (H,H) in Figure 1). When firms hold short positions, $f_{1,1}^i, f_{1,1}^j > 0$, the spot market becomes more competitive in that firms are credibly committing more production to period 1 (and less to period 2). This can be seen from condition (7): contracts increase firms' marginal revenues making them to behave more aggressively in the spot market at 1. In fact, if $f_{1,1}^i = f_{1,1}^j = a(1 - \delta)/2$, the perfectly competitive solution is implemented.¹⁴ Conversely, if firms take long positions, i.e., $f_{1,1}^i, f_{1,1}^j < 0$, the spot market becomes less competitive; and when $f_{1,1}^i = f_{1,1}^j = -a(1 - \delta)/4$, the monopoly solution is implemented.¹⁵

Obviously, in equilibrium firms do not trade any arbitrary amount of forwards. In deciding how many contracts to buy/sell, firm i 's evaluates the following payoff¹⁶

$$V_1^i = W_1^i(f_{1,1}^i, f_{1,1}^j)$$

where $W_1^i(f_{1,1}^i, f_{1,1}^j)$ are the spot (subgame-perfect) profits. There are no payments at the forward stage, only writing of contracts. Rearranging terms, firm i 's overall profits as a function of $f_{1,1}^i$ and $f_{1,1}^j$ can be written as

$$V_1^i = (p_{1,1}^f - p_1^s)f_{1,1}^i + p_1^s q_1^i(f_{1,1}^i, f_{1,1}^j) + \delta p_2^s q_2^i(s_2^i(f_{1,1}^i, f_{1,1}^j), s_2^j(f_{1,1}^i, f_{1,1}^j))$$

¹³Note that if for some reason firms' forward positions are long enough that $a + 2f_{1,1}^i - f_{1,1}^j < 0$, firms will produce nothing in period 1 and close their positions at the choke price a (this also applies to the second term in (8)).

¹⁴The competitive allocations when exhaustion takes two periods are $q_1^* = [a(1 - \delta) + 2\delta s_1]/2(1 + \delta)$ and $q_2^* = s_1 - q_1^*$. Note that because of scarcity rents, firms do not need to be fully contracted to implement the competitive solution.

¹⁵The monopoly allocations are $q_1^m = [a(1 - \delta) + 4\delta s_1]/4(1 + \delta)$ and $q_2^m = s_1 - q_1^m$.

¹⁶Note that under physical commitments, the term $p_{1,1}^f f_{1,1}^i$ would enter in i 's payoff because the contract revenue is explicitly pocketed at the forward stage.

where $p_t^s = p_t^s(q_t^i(f_{1,1}^i, f_{1,1}^j) + q_t^j(f_{1,1}^i, f_{1,1}^j))$ for $t = 1, 2$. As in Allaz and Vila (1993), the arbitrage payoff $(p_{1,1}^f - p_1^s)f_{1,1}^i$ is zero. Speculators and/or consumers share the same information as producers and therefore compete for forwards until $p_{1,1}^f = p_1^s$, where p_1^s is the expected period-1 spot price that is a function of forward quantities $f_{1,1}^i$ and $f_{1,1}^j$. Thus, firms are left with the contract-coverage dependent Cournot profit from the two periods, $p_1^s q_1^i + \delta p_2^s q_2^i$.¹⁷

Continuing with the case in which $q_1^i(f_{1,1}^i, f_{1,1}^j) + q_2^i(f_{1,1}^i, f_{1,1}^j) = s_1^i$, firm i 's best response to j 's forward position is

$$G_{1,1}^i(f_{1,1}^j) = \frac{a}{4}(1 - \delta) - \frac{1}{4}f_{1,1}^j \quad (9)$$

which leads to the equilibrium forward sales

$$f_{1,1}^i = f_{1,1}^j = \frac{a}{5}(1 - \delta), \quad (10)$$

period-1 equilibrium output

$$q_1^i = \frac{1}{3(1 + \delta)} \left(\frac{6}{5}a(1 - \delta) + 3\delta s_1^i \right) \quad (11)$$

and period-2 equilibrium output $q_2^i = s_2^i = s_1^i - q_1^i$.

The mere opportunity of trading forward has created a prisoner's dilemma for the two firms bringing them closer to competitive pricing (they still take positions below the competitive mark $a(1 - \delta)/2$). Forward trading makes both firms worse off relative to the case in which they stay away from the forward market. If firm j does not trade any forwards, then firm i has all the incentives to make forward sales (i.e., $f_{1,1}^i > 0$) as a way to allocate a larger fraction of its total stock s_1^i to the first period, which is the most profitable of the two (recall that $p_1^s > \delta p_2^s$). In the reproducible commodity (Cournot) game, forward trading allows a firm to capture Stackelberg profits—given that the other firm has not sold any forwards—by credibly committing in advance to

¹⁷While Allaz and Vila (1993) outline the forward trading stage that justifies these conclusions, it may be useful to explain why the mere existence of oligopoly rents gives rise to the contract market. Consider a pure-spot Cournot oligopolist who is approached by a third-party with a proposal of a bilateral delivery contract to be signed before the spot market. If the contract is observable and enforceable, it is a credible commitment to serve part of the demand (the third party can deliver the contracted amount to the spot market), so only some residual demand is left for the spot competition. Since the contract creates rents by increasing the profits of the oligopolist, the third party and the oligopolist can always find an agreement that profits both. This way there will be entry to the "commitment market" until these rents are dissipated. If each entrant can provide unlimited commitment, it takes only two speculators to arbitrate away the rents in the contract market.

the Stackelberg production. In our depletable-stock game, forward trading allows a firm to capture Stackelberg profits by committing a larger fraction of its overall stock to the first period.

Note that the (symmetric) equilibrium characterized above is unique; any other equilibrium, if it existed, would require one firm, say i , serving only in period 1 and the other firm, j , serving in both periods. The latter can certainly happen for asymmetric forward positions; for example, if $f_{1,1}^i = a(1 - \delta)/4$ and $f_{1,1}^j = 0$. But these forward positions do not constitute an equilibrium since j 's best response to $f_{1,1}^i$ is not 0 but $G_{1,1}^j(f_{1,1}^i) > 0$. This "revision" lowers the spot price in period 1, which, in turn, makes i revise its forward position downward and j revise his upward until both positions converge to the equilibrium positions in (10) and with both firms serving in both periods.¹⁸

We can now extend our discussion to the case where firms can also take forward positions $f_{2,2}^i$ (later we will also allow for positions $f_{1,2}^i$). Working backwards and given stocks $s_2^i = s_2^j > 0$ and positions $f_{2,2}^i = f_{2,2}^j$, consider the spot subgame at $t = 2$. Firm i 's best response to an output $q_2^j \geq 0$ by firm j is given by

$$R_2^i(q_2^j, f_{2,2}^i, f_{2,2}^j) = \arg \max_{q_2^i} W_2^i = [p_2^s q_2^i + (p_{2,2}^f - p_2^s) f_{2,2}^i] \leq s_2^i$$

where W_2^i is firm i 's payoff in the spot market at 2. Solving, we obtain the period-2 equilibrium output

$$q_2^i(f_{2,2}^i, f_{2,2}^j) = \min \left\{ s_2^i, \frac{a + 2f_{2,2}^i - f_{2,2}^j}{3} \right\} \geq 0. \quad (12)$$

Unlike in period 1, it is evident from (12) that forward contracting may do nothing to period-2 spot competition if the remaining stocks s_2^i and s_2^j are small enough.

Lemma 1 *If $s_2^i = s_2^j \leq a/3$, firms hold any positions for period 2 (including no positions) that ensure exhaustion in period 2 (i.e., $q_2^i(f_{2,2}^i, f_{2,2}^j) = s_2^i$ for both i and j).*

The proof is simple. Take $s_2^i = s_2^j = a/3$ and suppose firm j sells no forwards for period 2 (i.e., $f_{2,2}^j = 0$) and ask what would be i 's optimal amount of contracting. It is evident that it is the amount that allows i to implement its Stackelberg outcome. But if

¹⁸The reason why the contracting stage does not introduce the multiplicity of equilibria introduced by the first-period production stage of Saloner (1987) is because firm i 's period-1 output is decreasing in $f_{1,1}^j$, so j has incentives to sell some forwards even if it believes i has contracted enough to implement its Stackelberg outcome. In other words, forward contracting provides partial commitment unlike irreversible production.

$s_2^i = a/3$ the best i can do is to produce $q_2^i = a/3$ and let j produce likewise; in other words, implement the Cournot outcome (and more so if $s_2^i = s_2^j < a/3$). Taking a long position, however, can be detrimental for i if it ends up selling less than s_2^i , that is, if $a + 2f_{2,2}^i - f_{2,2}^j < 3s_2^i$. Thus, in equilibrium firms hold any forward position (including no position) consistent with the equilibrium outcome $q_2^i(f_{2,2}^i, f_{2,2}^j) = s_2^i$.

The situation changes when $s_2^i = s_2^j > a/3$ because now firms have Stackelberg incentives to take short positions.

Lemma 2 *If $s_2^i = s_2^j > 2a/5$, firms hold short positions $f_{2,2}^i = f_{2,2}^j = a/5$ and sell only a fraction of their remaining stocks in period 2 (i.e., $q_2^i(f_{2,2}^i, f_{2,2}^j) = 2a/5 < s_2^i$ for both i and j).*

The proof, including the uniqueness of equilibrium, is in Allaz and Vila (1993). When stocks are sufficiently large the exhaustion restriction is no longer binding, so firms operate as if they were selling a reproducible good.

The most interesting case for our analysis is the one in which remaining stocks are neither too small nor too large such that firms operate in a Hotelling world where forward contracting does matter.

Lemma 3 *If $a/3 < s_2^i = s_2^j \leq 2a/5$, firms hold short positions for period 2 that ensure exhaustion in period 2 (i.e., $f_{2,2}^i \geq 3s_2^i - a > 0$ and $q_2^i(f_{2,2}^i, f_{2,2}^j) = s_2^i$ for both i and j).*

The minimum amount of contracting $3s_2^i - a$ is readily obtained from (12). The intuition is as follows. If for some reason, j take a short position below $3s_2^j - a$, firm i 's best response, in an effort to come closer to its period-2 Stackelberg outcome (i.e., produce s_2^i and let j produce $R_2^j(s_2^i) < s_2^j$), is to increase its forward position above $3s_2^i - a$ enough to ensure its exhaustion while forcing j to leave part of its stock on the ground. But this is clearly suboptimal for j . Here again firms face a prisoner's dilemma. It would be better for them to sell no contracts and produce $a/3 < s_2^i$ in period 2, but firms have no means to stay away from the forward market.

Now that we understand the Stackelberg rationale for forward contracting in a market where the total supply (i.e., stock) is fixed, we can extend our discussion to cover the case where firms can also take forward positions for period 2 in period 1 (i.e., $f_{1,2}^i$ positions) and where initial stocks can be of any size (we retain the symmetry of the problem). Since forward contracting affects spot competition either by shifting production across periods, when the exhaustion restriction is binding, or by adding more production, when

part of the stock is left on the ground, it will also alter the thresholds that define the regions where firms operate in equilibrium, namely, (C,C), (H,H) or (R,R). See Figure 1. It is useful to structure the discussion that follows around the computation of these two thresholds.

In computing the first threshold, the one that separates region (C,C) from (H,H), we start by finding the size of the initial stocks below which forward contracting plays no strategic role in equilibrium.

Lemma 4 *If $s_1^i = s_1^j \leq a(1-\delta)/3$, firms hold any positions (including no positions) that ensure exhaustion in period 1 (i.e., $q_1^i(f_{1,1}^i, f_{1,1}^j, f_{1,2}^i, f_{1,2}^j) = s_1^i$ for both i and j).*

The logic here follows that of Lemma 1. If j signs no forwards, there is nothing i can do to displace part of j 's production to period 2 since $MR_1^j \equiv a - 2s_1^j - s_1^i > \delta a$. And since everything is sold in period 1, there is no strategic role for forward contracting for period 2.

But as soon as s_1^i (and s_1^j) goes above $a(1-\delta)/3$, there are Stackelberg motives to sell forwards.

Lemma 5 *If $a(1-\delta)/3 < s_1^i = s_1^j \leq 2a(1-\delta)/5 \equiv s_1^{CH}$, firms hold short positions for period 1 enough to ensure exhaustion in period 1 (i.e., $f_{1,1}^i \geq 3s_1^i - a(1-\delta) > 0$ and $q_1^i(f_{1,1}^i, f_{1,1}^j) = s_1^i$ for both i and j) and no positions for period 2 (i.e., $f_{1,2}^i = 0$).*

Analogous to Lemma 3, the minimum amount of contracting $3s_1^i - a(1-\delta)$ is readily obtained from the second term in (8). Again, when stocks are relatively small both firms end up taking forward positions that allocate all the stocks to period 1, so there is no strategic reason to take a forward position for period 2.

Lemma 5 also indicates that $s_1^{CH} \equiv 2a(1-\delta)/5$ —the threshold that separates region (C,C) from (H,H)—is higher than in Figure 1, $a(1-\delta)/3$, because of the competitive pressure introduced by forward trading that forces firms to sell more in period 1. Thus, if stocks are above s_1^{CH} , firms will necessarily serve both periods in equilibrium; and if they are very large, firms will operate as if they were selling a reproducible good.

Lemma 6 *If $s_1^i = s_1^j > 29a/35 \equiv s_1^{HR}$, firms hold short positions for period 1 (i.e., $f_{1,1}^i = a/5$ for both i and j) and for period 2 (i.e., $f_{1,2}^i = f_{2,2}^i = a/7$ for both i and j) and sell a fraction of their stocks (i.e., $q_1^i = 2a/5$ and $q_2^i = 3a/7$, where $q_1^i + q_2^i = s_1^{HR} < s_1^i$).*

Again, the proof follows from Allaz and Vila (1993), as the overall capacity does not constrain sales. The threshold s_1^{HR} that separates region (H,H) from (R,R) is also higher

than in Figure 1 for the same competitive reasons. When stocks are above s_1^{HR} , the two spot markets become disconnected from each other in equilibrium. The spot market in period 2 is the more competitive of the two because it is preceded by two forward openings and firms take short positions in both.¹⁹ Note that in the general model, where there is an unlimited number of spot openings, spot markets are always intertemporally connected because firms have always the opportunity to attend the next spot market and obtain δa .

We are now ready to complete our characterization of the equilibrium when stocks are in region (H,H), which is the most relevant case for our analysis because it illustrates the basic workings of the general model.

Proposition 1 *If $s_1^{CH} < s_1^i = s_1^j \leq s_1^{HR}$, the SPE outcome for both i and j is given by*

$$(i) f_{1,1}^i - \delta f_{1,2}^i \equiv H_1^i = \frac{a}{5}(1 - \delta),$$

$$(ii) f_{1,2}^i \geq \frac{5}{1 + \delta} \left[s_1^i - \frac{4a}{5} \right],$$

$$(iii) f_{2,2}^i + f_{1,2}^i \equiv F_2^i \geq \frac{3}{1 + \delta} \left[s_1^i - \frac{a(11 - \delta)}{15} \right],$$

(iv) q_1^i as shown in (11), and (v) $q_2^i = s_2^i = s_1^i - q_1^i > 0$.

We first prove (i) and (iv). Let us work backwards from the spot market at $t = 1$. Given s_1^i , $f_{1,1}^i$ and $f_{1,2}^i$ (for both i and j), if firms anticipate that in equilibrium $s_2^i, s_2^j > 0$ and that these stocks will be exhausted in period 2, then, at the period-1 spot market, they will choose quantities that equalize marginal revenues, that is,

$$\begin{aligned} a - 2q_1^i - q_1^j + f_{1,1}^i &= \delta(a - 2q_2^i - q_2^j + f_{1,2}^i), \text{ or} \\ a - 2q_1^i - q_1^j + (f_{1,1}^i - \delta f_{1,2}^i) &= \delta(a - 2q_2^i - q_2^j) \end{aligned}$$

where $q_2^i = q_2^i(s_2^i, s_2^j) = s_2^i$ for both i and j . Therefore, the payoff-relevant variables in the forward subgame are not the individual positions $f_{1,1}^i$ and $f_{1,2}^i$ but the composite position $H_1^i = f_{1,1}^i - \delta f_{1,2}^i$. By the same backward induction arguments laid out before (see eqs. (9) and (10)), we obtain that in equilibrium firms will choose $f_{1,1}^i$ and $f_{1,2}^i$ so as to satisfy $H_1^i = a(1 - \delta)/5$. On the other hand, given composite positions H_1^i and H_1^j , the period-1 equilibrium output is given by the second term in (8) with $f_{1,1}^i$ replaced by H_1^i ; and

¹⁹Note also that because $p_1^s > \delta p_2^s$ firms and/or consumers have no incentives to store today's production for tomorrow's use.

plugging equilibrium positions $H_1^i = H_1^j = a(1 - \delta)/5$ gives (11). It is relatively open for firms how much to trade in the forward markets, as long as their composite position H_1^i satisfies $f_{1,1}^i - \delta f_{1,2}^i = a(1 - \delta)/5$. For example, firms could fully contract their period-2 deliveries (i.e., $f_{1,2}^i = q_2^i$) and simultaneously take short positions for period 1 equal to $f_{1,1}^i = a(1 - \delta)/5 + \delta q_2^i$.

Parts (ii) and (iii) of Proposition 1, however, put limits as to how much firms contract in equilibrium. To derive these limits, let us now work backwards from the spot market at $t = 2$. Given equilibrium positions H_1^i and H_1^j , remaining stocks are

$$s_2^i = \frac{1}{1 + \delta} \left[s_1^i - \frac{2a}{5}(1 - \delta) \right] \quad (13)$$

for both i and j . Thus, if firm i 's aggregate position for period 2 is $F_2^i = f_{1,2}^i + f_{2,2}^i$, the period-2 equilibrium output is given by expression (12), after replacing $f_{2,2}^i$ by F_2^i . Hence, the exhaustion of symmetric stocks at $t = 2$ requires $F_2^i + F_2^j \geq 6s_2^i - 2a$. Replacing s_2^i by its equilibrium value in (13) and maintaining symmetry (i.e., $F_2^i = F_2^j$), we obtain part (iii) of the proposition. We now move to the forward stage at $t = 2$. Given stocks $s_2^i = s_2^j$ and forward positions $f_{1,2}^i = f_{1,2}^j$, the equilibrium positions $f_{2,2}^i(f_{1,2}^i, f_{1,2}^j)$ will depend on firms' rational expectations as to whether exhaustion will occur at $t = 2$ or not. If past positions, i.e., $f_{1,2}^i$ and $f_{1,2}^j$, are such that exhaustion is not expected to occur, we can proceed as in Lemma 6 to show that period-2 equilibrium positions, in such reproducible-good setting, are given by

$$f_{2,2}^i(f_{1,2}^i, f_{1,2}^j) = \frac{a}{5} - \frac{1}{5}f_{1,2}^i - \frac{1}{5}f_{1,2}^j$$

and, thereby, that equilibrium aggregate positions are given by $F_2^i(f_{1,2}^i, f_{1,2}^j) = f_{1,2}^i + f_{2,2}^i(f_{1,2}^i, f_{1,2}^j)$ for both i and j . If, on the other hand, exhaustion is expected to occur, then, aggregate equilibrium positions are given by $F_2^i(f_{1,2}^i, f_{1,2}^j) + F_2^j(f_{1,2}^i, f_{1,2}^j) \geq 6s_2^i - 2a$. Based on these aggregate equilibrium responses for varying values of $f_{1,2}^i$ and $f_{1,2}^j$, we can obtain the period-2 equilibrium output as a function of period-1 forward positions as follows

$$q_2^i(f_{1,2}^i, f_{1,2}^j) = \min \left\{ s_2^i, \frac{2a}{5} + \frac{3}{5}f_{1,2}^i - \frac{2}{5}f_{1,2}^j \right\} \geq 0$$

Therefore, if at the opening of the forward market in period 1 firms anticipate that they will be exhausting at 2, then, their forward equilibrium positions must be such that $2a + 3f_{1,2}^i - 2f_{1,2}^j \geq 5s_2^i$. Again, replacing s_2^i by its equilibrium value in (13) and maintaining symmetry (i.e., $f_{1,2}^i = f_{1,2}^j$), we obtain part (ii) of the proposition. The intuition behind part (ii) is that firms will not take a position in period 1 that will not be able to credibly (i.e., sequentially rationally) undo in period 2.

The proof of part (v) —that firms will exhaust in period 2 if $s_1^i \in (s_1^{CH}, s_1^{HR}]$ — is directly obtained from the application of Lemmas 5 and 6. In regard to Lemma 6, note that if $s_1^i = s_1^j = s_1^{HR}$, Proposition 1 predicts the following equilibrium outcome: (i) $f_{1,1}^i - \delta f_{1,2}^i = a(1 - \delta)/5$, (ii) $f_{1,2}^i \geq a/7(1 + \delta)$, (iii) $F_2^i \geq 2a/7(1 + \delta) + a\delta/5(1 + \delta)$, (iv) $q_1^i = 2a(1 - \delta)/5(1 + \delta) + 29a\delta/35(1 + \delta)$ and (v) $q_2^i = s_1^{HR} - q_1^i$. There seems to be a discrepancy between the predictions of Proposition 1 and of Lemma 6 at the border s_1^{HR} . There is none, however, because right at the border, discounting plays no role in the intertemporal allocation of output in equilibrium (i.e., $MR_1^i = MR_2^i = 0$); hence, we have a corner solution that for purposes of computing the equilibrium outcome calls for $\delta = 0$.

Proposition 1 shows a multiplicity of equilibria in the amount of forward contracting that naturally raises a concern regarding the difficulty this may impose for computing equilibrium strategies in games with a large number of market openings, like in the general model. Fortunately, such multiplicity vanishes as we move to the more relevant cases. As already seen in Lemma 6, it does when the exhaustion restriction is not binding. But more importantly for our analysis, the multiplicity also disappears when the stock at the period of exhaustion is sufficiently small. Indeed, Lemma 1 shows that if $s_2^i \leq a/3$,²⁰ forwards positions for period 2 become strategically irrelevant (i.e., we can just let $f_{1,2}^i = f_{2,2}^i = 0$ without any equilibrium consequences). In the general model, where firms have always the opportunity to serve the next spot market and obtain δa , the stock at the period of exhaustion will necessarily fall below $a/3$; otherwise, firms will be receiving zero at the margin. Thus, this two-period analysis tells us that in the general model we can ignore forward sales to the very last spot market served by the two firms and work backwards from the next to the last period.

2.4 Equilibrium for asymmetric stocks

We now look at the case in which stocks are of different sizes. Letting firm j be the smaller of the two firms, we will study how the equilibrium in the two-period game changes as we let s_1^j move from $s_1^j = 0$ to the symmetric case $s_1^j = s_1^i$, where s_1^i is some fixed value in the interval $[a(1 - \delta)/2, a(11 - \delta)/15]$. Rather than looking at all possible asymmetries in Figure 1, we restrict attention to values of s_1^i that ensure no contracting for the second period but nevertheless capture the essence of the asymmetric problem.

²⁰Or, alternatively, if $s_1^i \leq a(11 - \delta)/15$ (see part (ii) of Proposition 1). Note also that $4a/5 > a(11 - \delta)/15$.

The case $s_1^j = 0$ is immediate. A monopolist (i.e., firm i) will never sign forward contracts because this would only introduce more competition to the spot market (selling forwards has the same competition effect as selling part of the stock to a fringe of competitive suppliers). Now, to understand how stock asymmetries affect the equilibrium path when both firms hold some initial stock, it is useful to recall what firms seek to implement through forward contracting: if one firm does not sell forwards, the other can achieve Stackelberg profits by entering the forward market. Consider first the Stackelberg outcome for the larger firm. Firm i 's first-best is to implement $q_1^j = s_1^j$ and $q_2^j = 0$, i.e., it is optimal for i to let j exhaust in period 1, if

$$s_1^j \leq \frac{1}{4}a(1 - \delta). \quad (14)$$

Thus, when j is small enough, i will let j to sell only to the more profitable first period, even if i could commit part of its sales before j takes any action.²¹

Consider now firm j 's Stackelberg outcome. If allowed to move first, j would like to sell its entire stock in the first period as long as

$$s_1^j \leq \frac{1}{2}a(1 - \delta). \quad (15)$$

It is intuitively clear that when we consider j 's own stock, j 's first-best (i.e., Stackelberg) threshold for leaving stock for the less profitable second period is larger than in (14).

These inequalities imply that both firms prefer j 's early exhaustion in period 1 when j is small enough, i.e., (14) holds, and then there are no incentives to contract by either firm. However, j can extend its commitment to sell early by using the forward market even when its stock exceeds the level identified by (14).

Proposition 2 *If*

$$\frac{1}{4}a(1 - \delta) < s_1^j \leq \frac{5 - 2\sqrt{2}}{5}a(1 - \delta) \equiv s_1^{ST},$$

firm j implements its Stackelberg (i.e., first-best) solution: There is a two-period equilibrium where i does not contract at all (i.e., $f_{1,1}^i = 0$) and j commits to sell only in period 1 by contracting $f_{1,1}^j$ according to

$$f_{1,1}^j \geq f_{\min}(s_1^j) \equiv \frac{4}{3}s_1^j - \frac{1}{3}a(1 - \delta)$$

²¹The proof is immediate and, therefore, ignored here. Set $f_{1,1}^j = 0$ and solve for i 's best response in the forward market and then use the chosen position to find the equilibrium deliveries. Alternatively, one can change the timing in the pure spot market model to find the Stackelberg allocations.

Proof. See the Appendix.²² ■

Proposition 2 says that j needs to contract at least $f_{\min}(s_1^j)$ to achieve its first-best. Note that contracting incentives just disappear, i.e., $f_{\min}(s_1^j) = 0$, when $s_1^j = a(1 - \delta)/4$. Note also that if j contracts nothing when its stock is above the threshold in (14), i could achieve its first-best by taking a short position that would shift part of j 's sales to period 2. But j can prevent this by making the spot market in $t = 1$ less profitable to i through its own contracting —minimum contracting $f_{\min}(s_1^j)$ is calculated as a position that keeps i unwilling to sign contracts. Contracting more than $f_{\min}(s_1^j)$, e.g., $f_{1,1}^j = s_1^j$, is more than enough to keep i away from the forward market until (15) holds as an equality.

When j 's stock is above the Stackelberg threshold s_1^{ST} in Proposition 2, i 's first-best is to make j to deliver also in period 2 by taking a short position for period 1. Then, firm j contracts according to (9), i.e., $G_{1,1}^j(f_{1,1}^i) > 0$, which leads j to delivering in both periods. But if j is expected to deliver in both periods, firm i 's best contracting response must also be given by (9). Therefore, when both firms are active in both periods the only possible equilibrium is that both firms take position $a(1 - \delta)/5$ in the forward stage.²³

This two-period model illustrates how asymmetries can help firms to escape the competitive pressure introduced by the forward market. In fact, the smaller firm can greatly benefit from the forward market in that it may be able to implement its Stackelberg solution (unlike the larger firm which has nothing to gain from the opening of the forward market). Another way to appreciate how such stock asymmetries can suppress the pro-competitive effect of forward trading is to ask what would be the equilibrium consequences of adding a forward market before the opening of the spot markets, say, at $t = 0$. (this is important in a more general setting where spot markets in the distant future are preceded by a potentially large number of forward openings). Unlike in the symmetric case, there would be no consequences: $f_{0,1}^j = f_{\min}(s_1^j)$ and $f_{0,1}^i = f_{1,1}^i = f_{1,1}^j = 0$.

²²Note that $s_1^{ST} = 0.434a(1 - \delta)$.

²³Note that the symmetric contracting equilibrium extends below the threshold s_1^{ST} in Proposition 2. In fact, for $s_1^j \in [2a(1 - \delta)/5, s_1^{ST}]$ both equilibria coexist (and with one in mixed strategies) but the asymmetric equilibrium Pareto dominates (i.e., better for both firms) the symmetric one. Likewise, the asymmetric equilibrium extends above s_1^{ST} up to the threshold $a(1 - \delta)/2$ in (15); within this range there is no Pareto ranking of equilibria, however. In any case, this multiplicity is specific to the two-period setting and is inconsequential more generally because even small asymmetries in initial stocks will generate large asymmetries in the future as the smaller firm exhausts its stock (in Liski and Montero (2009) we formally show how this multiplicity disappears in a three-period model where the smaller firm is active in the first two periods). Furthermore, this multiplicity (note also that $s_1^{ST} > s_1^{CH}$) complicates, at least in some regions, the redrawing of Figure 1 when both spot and forward trading are present.

The two-period model also illustrates how forward contracting reinforces the fact that asymmetric firms will generally exit the market at different times. It expands the stock threshold for which firm j would exit the market after the first period from $s_1^j \leq a(1-\delta)/3$ —the threshold under pure-spot trading (see Figure 1)—to $s_1^j \leq s_1^{ST}$. This is because forward contracting makes the small firm to increase its deliveries to earlier periods (to $t = 1$ in this two-period model) while the large firm to do the same to later periods (where the smaller firm is absent).²⁴

Finally, it is interesting to point out that introduction of firms' asymmetries in the resource model has a different effect than in the reproducible-good model of Allaz and Vila (1993). Asymmetries in the reproducible-good model come from considering firms of different marginal cost of production with the "large" firm being the one with lower costs.²⁵ As shown in the Appendix, it is now the large firm the one that can greatly benefit from the forward market. In fact, when the cost difference is large enough the large firm can use the forward market to credibly implement its Stackelberg solution that is to fully displace the small firm from the market.²⁶

3 General model

There are $t = 1, \dots, N < \infty$ periods; each with a forward market opening followed by a spot market opening. At the forward stage in t , firms simultaneously decide the (additional) forward positions they will take for each and every future spot market, i.e., $\mathbf{f}_t^i = (f_{t,t}^i, f_{t,t+1}^i, \dots, f_{t,N}^i)$ for both i and j . Likewise, at the spot market in t firms simultaneously decide their (non-negative) deliveries q_t^i and q_t^j .

The history of the game after $t - 1$ periods (i.e., at the beginning of period t) is denoted by h_t and composed by the initial stocks, s_1^i and s_1^j , and the sequence of forward positions and deliveries, $(\mathbf{f}_1^i, \mathbf{f}_1^j, \dots, \mathbf{f}_{t-1}^i, \mathbf{f}_{t-1}^j)$ and $(q_1^i, q_1^j, \dots, q_{t-1}^i, q_{t-1}^j)$, respectively. Given

²⁴To see the latter consider any s_1^j such that under pure-spot trading firm j would serve both periods but that with forward trading would only serve period 1. Firm i 's deliveries in period 1 under pure-spot-trading and with forward-trading are, respectively, $\tilde{q}_1^i = [a(1-\delta) + 3\delta s_1^i]/3(1+\delta)$ and $q_1^i = [a(1-\delta) + 2\delta s_1^i - s_1^j]/2(1+\delta)$. Then, $q_1^i < \tilde{q}_1^i$ (and $q_2^i > \tilde{q}_2^i$) iff $s_1^j > a(1-\delta)/3$, which precisely indicates the range where j serves both periods in pure-spot equilibrium.

²⁵A lower marginal cost in the reproducible-good model implies a larger market share in the pure-spot (Cournot) game. Similarly, a larger stock in the resource model implies a lower opportunity cost of selling today instead of tomorrow.

²⁶de Frutos and Fabra (2009) also find that firms' asymmetries affect the way forward contracting influence the degree of competition in the spot market.

these sequences, remaining stocks at the end of $t - 1$ are equal to $s_t^i = s_1^i - \sum_{k=1}^{t-1} q_k^i$ for both i and j , and aggregate forward holdings for future spot markets are equal to $\mathbf{F}_{t-1}^i = (F_t^i(t-1), F_{t+1}^i(t-1), \dots, F_N^i(t-1))$ for both i and j , where $F_h^i(t) = \sum_{k=1}^t f_{k,h}^i$ is the aggregate position that firm i holds at t for the spot market that opens at $h \geq t$. Thus, for example, $F_t^i(t) = F_t^i(t-1) + f_{t,t}^i$.

A forward-sale strategy for firm i is a collection of functions

$$\mathbf{f}^i = (\mathbf{f}_1^i(\cdot), \dots, \mathbf{f}_N^i(\cdot))$$

where function $\mathbf{f}_t^i(h_t)$ specifies firm i 's (additional) forward positions as a function of the history h_t . A forward-strategy profile is denoted by $\mathbf{f} = (\mathbf{f}^i, \mathbf{f}^j)$.

A spot-sale strategy for firm i , on the other hand, is a collection of functions

$$\mathbf{q}^i = (q_1^i(\cdot), \dots, q_N^i(\cdot))$$

where function $q_t^i = q_t^i(h_t')$ specifies firm i 's output as a function of the history at the opening of the spot market in t , $h_t' = (h_t, \mathbf{f}_t^i, \mathbf{f}_t^j)$. A spot-sale strategy profile is denoted by $\mathbf{q} = (\mathbf{q}^i, \mathbf{q}^j)$. Thus, a strategy profile (\mathbf{f}, \mathbf{q}) together with the initial history $h_1 = (s_1^i, s_1^j, \mathbf{f}_0^i = \mathbf{f}_0^j = \mathbf{0})$ generate an outcome path that characterizes the development of the stocks, $s_{t+1}^i = s_t^i - q_t^i$, and of the contract coverages, $\mathbf{F}_t^i = \mathbf{F}_{t-1}^i + \mathbf{f}_t^i$, for both i and j .

Given some strategy profile (\mathbf{f}, \mathbf{q}) and history h_t' , firm i 's payoff at the spot stage in t is

$$W_t^i(h_t', \mathbf{f}, \mathbf{q}) = p_t^s(\cdot)q_t^i + \sum_{k=1}^t p_{k,t}^f f_{k,t}^i - p_t^s(\cdot)F_t^i(t) + \delta V_{t+1}^i(h_{t+1}, \mathbf{f}, \mathbf{q})$$

where $V_{t+1}^i(h_{t+1}, \mathbf{f}, \mathbf{q})$ is the continuation payoff at the forward stage in $t + 1$ and $h_{t+1} = (h_t', q_t^i, q_t^j)$. At the spot stage, firms not only attend the spot market but also close their forward positions at the spot price p_t^s . But as explained in the two-period model, forward contracts for period t are traded at the expected spot price at t ; hence, in equilibrium $p_{k,t}^f = p_t^s$ for all k (agents correctly anticipate the effect of forward positions on future spot prices, or more precisely, on firms' deliveries). In addition, since forward markets are only used to write contracts (i.e., to take positions), firm i 's payoff at the forward stage in t is simply

$$V_t^i(h_t, \mathbf{f}, \mathbf{q}) = W_t^i(h_t', \mathbf{f}, \mathbf{q}).$$

The strategy profile $(\hat{\mathbf{f}}, \hat{\mathbf{q}})$ constitutes a subgame-perfect equilibrium (SPE) if for any t and history h_t

$$\hat{\mathbf{f}}_t^i(h_t) = \arg \max_{\mathbf{f}_t^i} V_t^i(h_t, (\mathbf{f}^i, \hat{\mathbf{f}}^j), \hat{\mathbf{q}}) \quad (16)$$

and for any t and history h'_t

$$\hat{q}_t^i(h'_t) = \arg \max_{q_t^i} W_t^i(h'_t, \hat{\mathbf{f}}, (\mathbf{q}^i, \hat{\mathbf{q}}^j)) \quad (17)$$

where $\mathbf{f}^i = (\dots, \mathbf{f}_t^i, \hat{\mathbf{f}}_{t+1}^i(\cdot), \dots, \hat{\mathbf{f}}_N^i(\cdot))$ and $\mathbf{q}^i = (\dots, q_t^i, \hat{q}_{t+1}^i(\cdot), \dots, \hat{q}_N^i(\cdot))$.²⁷ Using backward induction we can construct the SPE strategies, which we have already done in Section 2 for the case $N = 2$. There, we also showed that the pay-off relevant history at the beginning of period $t = 1, 2$ was just the current state that consisted of remaining stocks, s_t^i and s_t^j , and existing aggregate positions, \mathbf{F}_{t-1}^i and \mathbf{F}_{t-1}^j , so we could safely write $h_t = (s_t^i, s_t^j, \mathbf{F}_{t-1}^i, \mathbf{F}_{t-1}^j)$. We now extend the analysis to show that the same holds for any $N > 2$.²⁸ However, we restrict attention to the symmetric case, which we can solve explicitly.

3.1 Symmetric equilibrium

Assume that initial stocks are the same, that is, $s_1^i = s_1^j$. If stocks are too large in that they do not affect the equilibrium outcome, equilibrium deliveries follow the Allaz and Vila's (1993) rule for reproducible goods, which is

$$q_t^i = q_t^j = \frac{t}{3 + 2t} a$$

where $t = 1, \dots, N$. Supplies increase and thus prices decline over time, so there are no incentives to store the good either. We rather want to focus in equilibria that are not constrained by the length of the game, i.e., where initial stocks imply exhaustion before the final stage N (in terms of our two-period model, we want to avoid be in the (R,R) region). Therefore, we assume stocks are insufficient to support the Allaz and Vila (1993) outcome:

$$s_1^i = s_1^j < \sum_{t=1}^N \frac{t}{3 + 2t} a.$$

Furthermore, we assume that stocks are not that small either that they will be exhausted in just one period (see Section 2).

²⁷Note our abuse of notation: the argmax operator in (16) returns a vector of positions not a single argument.

²⁸The two-period game serves in turn to define the continuation payoff for a game with three periods. Proceeding in this manner for any $N > 2$ we can establish the structure of the equilibrium strategies, as done in Appendix for the symmetric equilibrium. Note the similarity of this procedure to that in Kahn (1986).

Proposition 3 Consider $N < \infty$. Let T be the equilibrium stock-depletion time, where $2 \leq T < N$. Then, the symmetric SPE deliveries are given by

$$q_t^i = q_t^j = \left\{ \frac{a}{3} \left[\sum_{h=0}^{n-2} \delta^h - (n-1)\delta^{n-1} \right] \left[1 + \frac{t}{3+2t} \right] + \delta^{n-1} s_t^i \right\} \frac{1}{\sum_{h=0}^{n-1} \delta^h} \quad (18)$$

where $t = 1, 2, \dots, T-1$ and $n = n(t) = T - t + 1$ (number of periods to reach exhaustion in equilibrium). Furthermore, $q_T^i = q_T^j = s_1^i - \sum_{t=1}^{T-1} q_t^i$ and $q_t^i = q_t^j = 0$ for all $t > T$.

Proof. See the Appendix. ■

We start the proof by finding the equilibrium solution for the last three periods before exhaustion (i.e., $T-2, T-1$ and T). Based on this solution we then formulate an induction hypothesis specifying deliveries and contracting patterns for any period $T > t \geq 1$. Equilibrium deliveries are determined by best responses in the forward and spot stages of each period as dictated by (16) and (17).

From (16), we have that at period t the equilibrium contracting choice $f_{t,t}^i$ satisfies $\partial V_t^i(\cdot) / \partial f_{t,t}^i = 0$, that is

$$\begin{aligned} & \left\{ \frac{\partial p_t^s}{\partial q_t^i} (q_t^i - F_t^i(t-1)) + p_t^s + \delta \frac{\partial}{\partial s_{t+1}^i} V_{t+1}^i \frac{\partial s_{t+1}^i}{\partial q_t^i} \right\} \frac{\partial q_t^i}{\partial f_{t,t}^i} + \\ & \left\{ \frac{\partial p_t^s}{\partial q_t^j} (q_t^j - F_t^j(t-1)) + \delta \frac{\partial}{\partial s_{t+1}^j} V_{t+1}^j \frac{\partial s_{t+1}^j}{\partial q_t^j} \right\} \frac{\partial q_t^j}{\partial f_{t,t}^i} = 0 \end{aligned}$$

for both i and j . The first line is the pro-competitive effect of forward contracting showing how firm i 's own overall contracting tends to shift production to the present, and the second line is the strategic investment effect capturing firm i 's incentive to push firm j away from the more profitable current market.

In equilibrium, however, firms use each remaining opportunity at period t to make additional contract sales for future spot markets as long as there is stock left for delivery in those spot markets. These contracting choices at t for future spot markets satisfy

$$\frac{\partial V_t^i}{\partial f_{t,k}^i} + \frac{\partial V_{t+1}^i}{\partial f_{t,k}^i} + \dots + \frac{\partial V_k^i}{\partial f_{t,k}^i} = 0$$

for all k , where $T \geq k > t$. Firms have an incentive to take positions for k due to the so-called prisoner's dilemma effect introduced by the mere existence of the forward markets. But in this framework, future contracting has spillovers to the present: heavy contracted future spot markets tend to increase the relative profitability of earlier spot markets, influencing how each firm decides how to allocate its stock across the spot markets. Current deliveries are thus influenced by future positions in potentially complicated ways.

In the Appendix we show that, due to linearities in cost and demand, best responses for spot quantities at t depend only on the composite contract position held at t

$$H_t^i = \sum_{h=0}^{n-2} \delta^h F_t^i(t) - \delta^{n-1} \sum_{h=1}^{n-2} F_{t+h}^i(t)$$

with $n = T - t + 1$.²⁹

On the other hand, from (17) we obtain that equilibrium quantities at t satisfy

$$\frac{\partial p_t^s}{\partial q_t^i} (q_t^i - F_t^i) + p_t^s = \delta \frac{\partial V_{t+1}^i}{\partial s_{t+1}^i} \quad (19)$$

for i and j . Solving for both i and j (see the Appendix) leads to

$$q_t^i = \left[\frac{a}{3} \sum_{h=0}^{n-2} (\delta^h - (n-1)\delta^{n-1}) + \delta^{n-1} s_t^i + \frac{2}{3} H_t^i - \frac{1}{3} H_t^j \right] \frac{1}{\sum_{h=0}^{n-1} \delta^h}. \quad (20)$$

It is clear from (20) that future contracting influences spot deliveries through the composite H_t^i and not through individual positions. Thus, contracting best responses reduce to the choice of H_t^i , which in turn depends on the current forward choice \mathbf{f}_t^i . We find that in (symmetric) equilibrium H_t^i takes the form

$$H_t^i = \frac{t}{3 + 2t} a \sum_{h=0}^{n-2} (\delta^h - (n-1)\delta^{n-1}) \quad (21)$$

with $n = T - t + 1$. This latter expression together with the quantity best-responses in (20) lead to the symmetric equilibrium deliveries in Proposition 3.

To illustrate the effect of forward contracting in this more general setting, it helps to contrast the delivery rule in Proposition 1 with the one in the equivalent pure-spot (symmetric) game. Suppose, for example, that it takes three periods to exhaust in pure-spot equilibrium. For this to be the case, it must hold

$$a - 2q_1^i - q_1^j = \delta(a - 2q_2^i - q_2^j), \quad (22)$$

$$a - 2q_2^i - q_2^j = \delta(a - 2q_3^i - q_3^j), \text{ and} \quad (23)$$

$$(q_1^i + q_2^i + q_3^i) = s_1^i \quad (24)$$

for both i and j (marginal revenues are equalized in present value and stocks are fully depleted). These conditions lead to first-period deliveries

$$\tilde{q}_1^i = \tilde{q}_1^j = \left\{ \frac{a}{3} (1 + \delta - 2\delta^2) + \delta^2 s_1^i \right\} \frac{1}{(1 + \delta + \delta^2)}. \quad (25)$$

²⁹Note that in the Appendix H_t^i would be denoted as $H_t^i(t)$. More generally, in the appendix we will refer to $H_k^i(t)$ as the composite position held at t for the spot at $k \geq t$.

More generally, if \tilde{T} denotes the equilibrium stock-depletion time in this hypothetical pure-spot game (where $2 \leq \tilde{T} < N$), then the equilibrium delivery in period t is

$$\tilde{q}_t^i = \tilde{q}_t^j = \left\{ \frac{a}{3} \left[\sum_{h=0}^{\tilde{n}-2} \delta^h - (\tilde{n}-1)\delta^{\tilde{n}-1} \right] + \delta^{\tilde{n}-1} s_t^i \right\} \frac{1}{\sum_{h=0}^{\tilde{n}-1} \delta^h},$$

with $\tilde{n} = \tilde{T} - t + 1$.

Now, rewrite (18) to obtain

$$q_t^i = q_t^j = \left\{ \frac{a}{3} \left[\sum_{h=0}^{n-2} \delta^h - (n-1)\delta^{n-1} \right] + \delta^{n-1} s_t^i \right\} \frac{1}{\sum_{h=0}^{n-1} \delta^h} + \frac{1}{3} \frac{H_t^i}{\sum_{h=0}^{n-1} \delta^h} \quad (26)$$

where H_t^i is the equilibrium composite as defined in (21). Without forward markets, $H_t^i = 0$ and firm i 's delivery is, not surprisingly, equal to the pure-spot delivery. Thus, the last term in (26) captures exactly how contracting increases supplies in a given period relative to the pure-spot case. For example, if T is very large and we are two periods away from exhaustion ($n = 2$), then H_{T-1}^i is close to

$$\frac{1}{6}a(1 - \delta),$$

and deliveries $q_{T-1}^i = q_{T-1}^j$ to

$$\frac{1}{2(1 + \delta)}(a(1 - \delta) + 2\delta s_{T-1}^i),$$

which are equal to the socially efficient deliveries in the first period of a two-period model with initial stocks equal to $s_{T-1}^i = s_{T-1}^j$.

3.2 Continuous-time limit of the symmetric equilibrium

We just saw that the number of forward openings before any given spot market has an important effect on the degree of competition in that spot market. Thus, the size of the stocks, which determine how many times the market must open to exhaust the overall capacity, will influence the degree of competition. Alternatively, we can take the stocks as given and measure the frequency of transactions by the period length. We can always set the period length long enough that any initial stocks holdings are consumed in just two periods, so firms face the prisoners' dilemma from contracting only once. If we reduce the period length, depletion of the same initial holdings necessarily requires of more periods; in the limit when we let the period length vanish, the two-period model is transformed into a continuous-time version. In such a case, the firms face the prisoners' dilemma arbitrarily many times after any positive interval of time. Nevertheless, it is not

a priori clear if the overall capacity constraint puts a limit to this competitive pressure. We explore this next.

It proves useful to explain first how the period length can be incorporated into the pure-spot game. Let Δ denote the period length and suppose for now that it takes three periods to exhaust the initial holdings ($s_1^i = s_1^i$) in equilibrium. The corresponding equilibrium conditions are (22), (23) and

$$\Delta(q_1^i + q_2^i + q_3^i) = s_1^i$$

which lead to the same first-period delivery as in (25), except that the stock is scaled by the period length, i.e., s_t^i is replaced by s_t^i/Δ . More generally, if the symmetric pure-spot equilibrium lasts for \tilde{T} periods, then the equilibrium delivery at t is

$$\tilde{q}_t^i = \tilde{q}_t^j = \left\{ \frac{a}{3} \left[\sum_{h=0}^{\tilde{n}-2} \delta^h - (\tilde{n}-1)\delta^{\tilde{n}-1} \right] + \delta^{\tilde{n}-1} \frac{s_t^i}{\Delta} \right\} \frac{1}{\sum_{h=0}^{\tilde{n}-1} \delta^h}, \quad (27)$$

where $\tilde{n} = \tilde{T} - t + 1$. It is thus clear that the period length only scales the stock holding s_t^i . The same conclusion holds for deliveries in the forward-contracting equilibrium: the effect of contracts on deliveries, measured through H_t^i in (26), depends only on the number of forward openings, but not on how short or long these openings are. Therefore, the delivery rule (26) remains the same except that the stock holding s_t^i must be scaled by the period length as done in (27).

Fixing $\Delta > 0$, we can find a game long enough $N = N(\Delta)$ such that the stocks are exhausted in $T = T(\Delta) < N(\Delta) < \infty$ periods. As explained in the Appendix, the (scaled) delivery rule (26) implies a finite exhaustion time even if $\Delta \rightarrow 0$; we denote this exhaustion time by Υ . Also, we denote by τ the time elapsed after t periods, so $t = \tau/\Delta$.

Proposition 4 *Let $T(\Delta) < N(\Delta)$ for all $\Delta > 0$. As $\Delta \rightarrow 0$, the symmetric SPE deliveries become arbitrarily close to the socially efficient deliveries at any given time $\tau > 0$.*

Proof. See the Appendix. ■

In the Appendix we show that the continuous-time deliveries at $\Upsilon > \tau > 0$ are (r is the instantaneous interest rate)

$$q_\tau^i = q_\tau^j = \frac{a(e^{r(\Upsilon-\tau)} - 1 - r(\Upsilon - \tau))}{2(e^{r(\Upsilon-\tau)} - 1)} + \frac{r s_\tau^i}{e^{r(\Upsilon-\tau)} - 1}$$

which correspond to the socially optimal supply at τ for a given remaining stock s_τ^i .

Intuitively, as $\Delta \rightarrow 0$, it must be the case that $f_{t,t}^i \rightarrow 0$ soon after the start of the game: the cumulative contract positions \mathbf{F}_t^i and \mathbf{F}_t^j almost instantly converge to their equilibrium levels. The social optimality of spot actions, given \mathbf{F}_t^i and \mathbf{F}_t^j , requires

$$d[a - q_t^i - q_t^j - (q_t^i - F_t^i)]/d\Delta = r, \quad (28)$$

i.e., marginal revenues, after controlling for contract coverage, to grow at the rate of interest $r > 0$. Denoting uncovered deliveries by $u_t^i = q_t^i - F_t^i$, condition (28) can be rewritten as

$$dp_t^s/d\Delta - du_t^i/d\Delta = r \quad (29)$$

But from Proposition 4 we know that when $\Delta \rightarrow 0$ prices grow at the rate of interest, which implies

$$du_t^i/d\Delta = 0 \quad (30)$$

for both i and j . It then follows that in continuous time $u_\tau^i = 0$ for all $\tau > 0$, that is, firms are fully contracted as soon as \mathbf{F}_t^i and \mathbf{F}_t^j have converged to their equilibrium levels, which happens almost instantly when $\Delta \rightarrow 0$.³⁰

4 Concluding remarks

We found that forward contracting can have substantial implications for resource depletion in a non-cooperative oligopolistic environment. In fact, we showed for symmetric firms that the subgame-perfect equilibrium path approaches the perfectly competitive path as firms take an increasing number of periods to deplete their stocks. We also showed, however, that when firms have initial stocks of different sizes, they can credibly contain part of the competitive pressure of forward contracting. It is then a unique feature of the resource model that equilibrium contracting and the degree of competition depends on resource endowments. Although formally our asymmetric result comes from a two-period model, it follows from a simple principle and we see no reason for it not to hold in a more general setting, i.e., the forward contracting reinforces the fact that asymmetric firms exit the market at different times. In other words, the mere possibility of trading forwards leads, in equilibrium, to the smaller firm to exhaust earlier and the larger firm later relative to the pure-spot equilibrium outcome. We also showed this for

³⁰This also shows neatly why forward contracting cannot support the open-loop equilibrium of the pure-spot game. Firms become fully contracted only when the market reaches perfect competition.

a three-period model in our working paper (Liski and Montero, 2009).³¹

It is yet to be discussed whether and to what extent forward contracting could also affect the possibility of collusion in this market. Recall that for a reproducible-commodity market, Liski and Montero (2006) have already shown that forward trading increases the scope of collusion —independently of the form of competition— by allowing firms to either construct harsher punishments or limit the deviation profits. Whenever nonstationary and collusive strategies are feasible in a pure spot-trading equilibrium in the resource context, then forward contracting should make it easier to sustain them. In the resource model, collusive strategies can exist when the overall consumption horizon is infinite, for example, due to (high) stock-dependent extraction costs or an infinite choke price —the price at which demand falls to zero. But when the choke price is finite and there is a gap between this price and the cost of extracting the last unit, as in our model, the consumption horizon is finite (both under perfect competition and monopoly), stipulating additional modeling assumptions facilitating the construction of collusive strategies.³²

5 Appendix

5.1 Proof of Proposition 2

We derive $f_{\min}(s_1^j)$ as follows. First, we find the Stackelberg (i.e., first-best) payoff and deliveries for i (the larger firm), given that j is holding some contracts $f_{1,1}^j$. This defines the most i can achieve in the original game. Second, we find the contracting level $f_{1,1}^j$ that induces j , "the follower", to produce only in the first period. This will define $f_{\min}(s_1^j)$. Given this level of contracting by j , i can implement its first-best in the original game by taking no position and letting j to sell only to period 1. Third, we will derive the Stackelberg threshold s_1^{ST} , under which this characterization holds.

Consider the first-best choice of q_1^i . Given q_1^i and $f_{1,1}^j$, j 's best-response satisfies (recall

³¹In the working paper we also provide a discussion on how to build an approximate solution for this asymmetric case in the general model.

³²One may borrow an insight from the durable-good monopoly literature and ask whether firms could sustain a collusive path that only asymptotically approaches the choke price, very much in the spirit of Ausubel and Deneckere (1989) and Gul (1987). Such a collusive path is however harder to sustain here: in Ausubel and Deneckere (1989) and Gul (1987) the punishment path entails zero profits for firms (so it is always possible to fashion an asymptotic collusive path where the present value from colluding is greater than the one-shot deviation profit), whereas here the sellers receive a positive resource rent that may provide more surplus than the collusive plan, at least in the final part of the collusive plan.

that $s_1^i < s_1^{HR}$, which guarantees exhaustion in period 2)

$$a - 2q_1^j - q_1^i + f_{1,1}^j = \delta(a - (s_1^i - q_1^i) - (s_1^j - q_1^j)) - \delta(s_1^i - q_1^i),$$

which leads to

$$q_1^j(q_1^i, f_{1,1}^j) = (a(1 - \delta) + 2\delta s_1^j + f_{1,1}^j - (1 + \delta)q_1^i + \delta s_1^i) \frac{1}{2(1 + \delta)}.$$

On the other hand and given $f_{1,1}^j$, firm i 's first-best payoff is

$$\max_{q_1^i} \{p_1^s(q_1^i, q_1^j(q_1^i, f_{1,1}^j))q_1^i + \delta p_2^s(s_1^i - q_1^i, s_1^j - q_1^j(q_1^i, f_{1,1}^j))(s_1^i - q_1^i)\}.$$

Solving

$$q_1^i(f_{1,1}^j) = (a(1 - \delta) + 2\delta s_1^i - f_{1,1}^j) \frac{1}{2(1 + \delta)},$$

and evaluating the follower's best-response gives

$$q_1^j(q_1^i(f_{1,1}^j), f_{1,1}^j) = (a(1 - \delta) + 4\delta s_1^j + 3f_{1,1}^j) \frac{1}{4(1 + \delta)}.$$

Contracting $f_{\min}(s_1^j)$ is defined by

$$q_1^j(q_1^i(f_{1,1}^j), f_{1,1}^j) = s_1^j.$$

Finally, the Stackelberg threshold s_1^{ST} is found as follows. If the second term in (8) results in $q_1^j < s_1^j$ given (symmetric) contracting $f_{1,1}^i = f_{1,1}^j = a(1 - \delta)/5$, then, both firms could be active in equilibrium in period 2. This defines a lower threshold, i.e., $s_1^j > 2a(1 - \delta)/5$, for the symmetric contracting equilibrium to be valid. The proof follows that of Lemma 5. A second equilibrium is also possible when $s_1^j > 2a(1 - \delta)/5$. First, note that since (15) holds, j 's best-response to $f_{1,1}^i = 0$ is $f_{\min}(s_1^j)$. On the other hand, firm i 's best-response to $f_{\min}(s_1^j)$ is indeed $f_{1,1}^i = 0$ provided s_1^j is not too large (we omit the exact number here). Firm i strictly "prefers" this latter equilibrium if it leads to a payoff larger than that under the symmetric contracting equilibrium. Comparing i 's payoffs, we find this to be case if (to save space we do not report the payoff expressions)

$$s_1^j \leq \frac{5 - 2\sqrt{2}}{5}a(1 - \delta) \equiv s_1^{ST} = 0.434a(1 - \delta).$$

5.2 Allaz and Vila for asymmetric firms

Consider two firms (i and j) producing a reproducible commodity with marginal costs, respectively, c_i and c_j with $c_i < c_j$. Firm i is the "large firm" in the sense that it has

a larger market share in the pure-spot (Cournot) game. Firms attend the spot market by simultaneously choosing quantities q^i and q^j . The spot price p^s is given by the linear inverse demand function $p^s(q^i + q^j) = a - (q^i + q^j)$, with

$$c_j - c_i \equiv \Delta < a - c_j$$

so that both firms produce in pure-spot equilibrium. Before the opening of the spot market firms are also free to simultaneously take forward positions for the spot market that follows. Forward transactions are denoted by f^i and f^j and the forward price by p^f .

At the opening of the spot market, and given f^i and f^j , firm i solves

$$\max_{q^i} W^i = p^s(\cdot)q^i - c_i q^i + (p^f - p^s(\cdot))f^i$$

Using both first-order conditions, we find that the subgame-perfect spot quantities are given by

$$q^i(f^i, f^j) = \frac{a - 2c_i + c_j + 2f^i - f^j}{3} \quad (31)$$

$$q^j(f^i, f^j) = \frac{a - 2c_j + c_i + 2f^j - f^i}{3} \quad (32)$$

Anticipating the equilibrium outcome (31)–(32), at the forward stage firm i solves (arbitrage profits vanish, i.e., $p^f = p^s$)

$$\max_{f^i} W^i(q^i(f^i, f^j), q^j(f^i, f^j)) = p^s q^i(f^i, f^j) - c_i q^i(f^i, f^j)$$

with $p^f = p^s = p^s(q^i(f^i, f^j) + q^j(f^i, f^j))$. Applying the envelope theorem leads to

$$\frac{\partial p^f}{\partial q^i} \frac{\partial q^i}{\partial f^i} f^i + \frac{\partial p^f}{\partial q^j} \frac{\partial q^j}{\partial f^i} f^i + p^f + \frac{\partial p^s}{\partial q^j} \frac{\partial q^j}{\partial f^i} (q^j - f^i) - p^s = 0$$

which reduces to

$$-\frac{2}{3}f^i + \frac{1}{3}q^i(f^i, f^j) = 0.$$

Replacing $q^i(f^i, f^j)$, we obtain the (equilibrium) forward positions

$$\hat{f}^i = \frac{a - 3c_i + 2c_j}{5}$$

$$\hat{f}^j = \frac{a - 3c_j + 2c_i}{5}.$$

For \hat{f}^i and \hat{f}^j to be equilibrium positions we require $q^i(\hat{f}^i, \hat{f}^j) > 0$ and $q^j(\hat{f}^i, \hat{f}^j) > 0$. Since the restriction for j is more demanding, we actually require

$$q^j(\hat{f}^i, \hat{f}^j) = \frac{2}{5} [a - 3c_j + 2c_i] > 0$$

or

$$\Delta < \frac{a - c_j}{2} \quad (33)$$

Note that $\hat{f}^j < 0$ is the same as $q^j(\hat{f}^i, \hat{f}^j) < 0$, so we cannot have an equilibrium where j goes long in an effort to increase prices.

Let us now look at firm i 's Stackelberg outcome in the absence of forward trading. It is easy to show that it is Stackelberg optimal for i to leave j outside the market if $\Delta > (a - c_j)/2$. In such a case, i will produce $q^i = a - c_j$ leading to an equilibrium price equal to c_j . Therefore, when firms are sufficiently different, i.e., condition (33) does not hold, the opening of the forward market allows the large firm to implement its Stackelberg solution with a price a bit below c_j and leaving firm j outside the market. More precisely, whenever $a - c_j \geq \Delta \geq (a - c_j)/2$, firm i implements its Stackelberg outcome by selling $f^i = f_{\min}$ to just keep j away from the spot market (if $\Delta = (a - c_j)/2$ then $f_{\min} = \hat{f}^i$ and if $\Delta = a - c_j$ then $f_{\min} = 0$) with final quantities equal to

$$q^i(f^i = f_{\min}, f^j = 0) = a - c_j$$

$$q^j(f^i = f_{\min}, f^j = 0) = 0$$

Unlike in the exhaustible-resource case, here it is the large firm the one that strictly benefits from the opening of the forward market.

5.3 Proof of Proposition 3

The induction hypothesis specifies a structure for the equilibrium deliveries and contracting positions at some step $n \geq 2$ (recall that n indicates the number of periods before exhaustion in equilibrium; we deliberately use the term step instead of period to make clear that we are moving backwards in time: step 2 is two periods from exhaustion, step 3 is three periods from exhaustion, etc.). Assuming that such structure holds for n we then show that it also holds for $n + 1$. We arrive at the equilibrium hypothesis by explicitly solving equilibrium deliveries and positions for $n = 1, 2$ and 3.

5.3.1 Step $n = 1$

Since $T < N$ is the period of exhaustion (i.e., the last period of production), step $n = 1$ is simply period T , and thus $q_1^i = s_1^i$ for both i and j . Note that for this proof q_n^i and s_n^i denote firm i 's deliveries and stock at step n , respectively.

5.3.2 Step $n = 2$

Let $F_m^i(n)$ denote the contract position held at step n for the spot market at step m . The spot market at step $n = 2$ opens with positions $F_2^i(2)$, $F_2^j(2)$, $F_1^i(2)$ and $F_1^j(2)$. However, from our two-period model, we know we can set $F_1^i(2) = F_1^j(2) = 0$ without any loss of generality.³³ The spot best response for each firm i solves

$$\frac{\partial[p_2^s \cdot (q_2^i - F_2^i(2))]}{\partial q_2^i} + \delta \frac{\partial V_1^i}{\partial s_1^i} \frac{\partial s_1^i}{\partial q_2^i} = 0, \quad (34)$$

where $V_1^i = p_1^s q_1^i = p_1^s s_1^i$ from step $n = 1$. We can rewrite (34) as

$$a - 2q_2^i - q_2^j + F_2^i(2) - \delta(a - 2q_1^i - q_1^j) = 0.$$

that together with $q_1^i = s_2^i - q_2^i$ and $q_1^j = s_2^j - q_2^j$ yields the subgame-perfect deliveries

$$q_2^i = \left[\frac{1}{3}a(1 - \delta) + \delta s_2^i + \frac{2}{3}F_2^i(2) - \frac{1}{3}F_2^j(2) \right] \frac{1}{1 + \delta}. \quad (35)$$

for both i and j .

Consider now the choice of $F_2^i(2)$ in the forward stage of step $n = 2$ given the previously taken position $F_2^i(3)$. Note that $F_2^i(2) = F_2^i(3) + f_{2,2}^i$, where $f_{2,2}^i$ is the additional contracting at $n = 2$. In this proof we can consider the choice $F_2^i(2)$ directly. This choice satisfies:

$$\begin{aligned} & \left\{ \frac{\partial[p_2^s \cdot (q_2^i - F_2^i(3))]}{\partial q_2^i} + \delta \frac{\partial V_1^i}{\partial s_1^i} \frac{\partial s_1^i}{\partial q_2^i} \right\} \frac{\partial q_2^i}{\partial F_2^i(2)} \\ & + \left\{ \frac{\partial[p_2^s \cdot (q_2^j - F_2^j(3))]}{\partial q_2^j} + \delta \frac{\partial V_1^j}{\partial s_1^j} \frac{\partial s_1^j}{\partial q_2^j} \right\} \frac{\partial q_2^j}{\partial F_2^i(2)} = 0 \end{aligned}$$

for both i and j . Using $\partial q_2^i / \partial F_2^i(2) = 2/3(1 + \delta)$ and $\partial q_2^j / \partial F_2^j(2) = -1/3(1 + \delta)$ from (35), we obtain

$$[a - 2q_2^i - q_2^j + F_2^i(3) - \delta(a - 2q_1^i - q_1^j)] \frac{2}{3(1 + \delta)} + [F_2^i(3) - q_2^i - \delta q_1^i] \frac{-1}{3(1 + \delta)} = 0$$

The above implies that subgame-perfect deliveries, as a function of previously taken positions, can be expressed as

$$q_2^i = \left[\frac{2}{5}a(1 - \delta) + \delta s_2^i + \frac{3}{5}F_2^i(3) - \frac{2}{5}F_2^j(3) \right] \frac{1}{1 + \delta}. \quad (36)$$

Equating (35) and (36) yields forward equilibrium responses for each firm i

$$F_2^i(2) = \frac{1}{5}(1 - \delta)a + \frac{4}{5}F_2^i(3) - \frac{1}{5}F_2^j(3). \quad (37)$$

We have just characterized two-period equilibrium deliveries and forward positions as a function of previous positions $F_2^i(3)$ and $F_2^j(3)$.

³³See the last sentence of Section 2.3.

5.3.3 Step $n = 3$

The spot market at step $n = 3$ opens with forward positions taken for spot markets at $n = 3$ and $n = 2$, i.e., $F_3^i(3)$ and $F_2^i(3)$ for both i and j . Firm i 's spot best response depends on them as follows

$$\frac{\partial[p_3^s \cdot (q_3^i - F_3^i(3))]}{\partial q_3^i} + \delta \frac{\partial V_2^i(s_2^i, s_2^j, F_2^i(3), F_2^j(3))}{\partial s_2^i} \frac{\partial s_2^i}{\partial q_3^i} = 0.$$

which, for our functional forms, reduces to

$$a - 2q_2^i - q_2^j + F_3^i(3) - \delta \frac{\partial V_2^i(s_2^i, s_2^j, F_2^i(3), F_2^j(3))}{\partial s_2^i} = 0. \quad (38)$$

To find an expression for $\partial V_2^i(\cdot)/\partial s_2^i$, note that

$$V_2^i = (a - q_2^i - q_2^j)(q_2^i - F_2^i(3)) + \delta(a - q_1^i - q_1^j)q_1^i + C \quad (39)$$

where C is just a constant that stands for the open financial positions (see expressions V_t^i and W_t^i in the main text). From (36) we have that $\partial q_2^i/\partial s_2^i = \delta/(1 + \delta)$, and since $q_1^i = s_2^i - q_2^i$, we also have that $\partial q_1^i/\partial s_2^i = 1/(1 + \delta)$. Using these expressions we differentiate V_2^i with respect to s_2^i to obtain

$$\partial V_2^i(\cdot)/\partial s_2^i = \frac{\delta}{1 + \delta} (2a - 2s_2^i - s_2^j + F_2^i(3)). \quad (40)$$

Combining (38) and (40) yields

$$q_3^i = \left\{ \frac{a}{3}(1 + \delta - 2\delta^2) + \delta^2 s_3^i + \frac{2}{3}[(1 + \delta)F_3^i(3) - \delta^2 F_2^i(3)] - \frac{1}{3}[(1 + \delta)F_3^j(3) - \delta^2 F_2^j(3)] \right\} \frac{\delta^2}{1 + \delta + \delta^2} \quad (41)$$

which can be conveniently rewritten as

$$q_3^i = \left[\frac{a}{3}(1 + \delta - 2\delta^2) + \delta^2 s_3^i + \frac{2}{3}H_3^i(3) - \frac{1}{3}H_3^j(3) \right] \frac{1}{1 + \delta + \delta^2} \quad (42)$$

where $H_3^i(3) = (1 + \delta)F_3^i(3) - \delta^2 F_2^i(3)$. As explained in the main text, the spot response to contracts depends on the composite $H_3^i(3)$, not the individual positions. More generally, we let $H_m^i(n)$ denote the forward composite that firm i holds at step n for spot markets at steps $m, m - 1, \dots, 2$.

We can now consider the (simultaneous) choices of $F_3^i(3)$ and $F_2^i(3)$ at step $n = 3$, given previously taken positions $F_3^i(4)$, $F_2^i(4)$, $F_3^j(4)$ and $F_2^j(4)$. The choice of $F_3^i(3)$

satisfies

$$\left[\left\{ \frac{\partial[p_3^s \cdot (q_3^i - F_3^i(4))]}{\partial q_3^i} + \delta \frac{\partial V_2^i}{\partial s_2^i} \frac{\partial s_2^i}{\partial q_3^i} \right\} \frac{\partial q_3^i}{\partial H_3^i(3)} \right] \quad (43)$$

$$+ \left\{ \frac{\partial[p_3^s \cdot (q_3^j - F_3^j(4))]}{\partial q_3^j} + \delta \frac{\partial V_2^j}{\partial s_2^j} \frac{\partial s_2^j}{\partial q_3^j} \right\} \frac{\partial q_3^j}{\partial H_3^j(3)} \left[\frac{\partial H_3^i(3)}{\partial F_3^i(3)} = 0 \right] \quad (44)$$

that provided that $\partial H_3^i(3)/\partial F_3^i(3) \neq 0$ simplifies to

$$\left[\left\{ a - 2q_3^i - q_3^j + F_3^i(4) - \delta \frac{\partial V_2^i(s_2^i, s_2^j, F_2^i(4), F_2^j(4))}{\partial s_2^i} \right\} \frac{2}{3} \right. \\ \left. + \left\{ F_3^i(4) - q_3^i - \delta \frac{\partial V_2^i(s_2^i, s_2^j, F_2^i(4), F_2^j(4))}{\partial s_2^j} \right\} \frac{-1}{3} \right] = 0.$$

An expression for $\partial V_2^i(\cdot)/\partial s_2^i$ can be readily obtained from (40) but with $F_2^i(3)$ replaced by $F_2^i(4)$. To obtain an expression for $\partial V_2^i(\cdot)/\partial s_2^j$, replace $F_2^i(3)$ in (39) by $F_2^i(4)$, and differentiate with respect to s_2^j noting that $\partial q_2^j/\partial s_2^j = \delta/(1+\delta)$ and $\partial q_1^j/\partial s_2^j = 1/(1+\delta)$.

This gives

$$\partial V_2^i(\cdot)/\partial s_2^j = \frac{\delta}{1+\delta} (F_2^i(4) - s_2^j). \quad (45)$$

Substituting into the first-order condition (43) for both $F_3^i(3)$ and $F_3^j(3)$ and solving for i and j yields:

$$q_3^i = \left\{ \frac{2a}{5} (1 + \delta - 2\delta^2) + \delta^2 s_3^i + \frac{3}{5} [(1 + \delta)F_3^i(4) - \delta^2 F_2^i(4)] \right. \\ \left. - \frac{2}{5} [(1 + \delta)F_3^j(4) - \delta^2 F_2^j(4)] \right\} \frac{1}{1 + \delta + \delta^2}. \quad (46)$$

Equate now (41) and (46), for both i and j , to obtain the forward best response for each firm i

$$F_3^i(3) = \left\{ \frac{a}{5} (1 + \delta - 2\delta^2) + \delta^2 F_2^i(3) + \frac{4}{5} [(1 + \delta)F_3^i(4) - \delta^2 F_2^i(4)] \right. \\ \left. - \frac{1}{5} [(1 + \delta)F_3^j(4) - \delta^2 F_2^j(4)] \right\} \frac{1}{1 + \delta}. \quad (47)$$

Multiply next both sides by $1 + \delta$ and rearrange to obtain

$$(1 + \delta)F_3^i(3) - \delta^2 F_2^i(3) \equiv H_3^i(3) = \frac{a}{5} (1 + \delta - 2\delta^2) + \frac{4}{5} H_3^i(4) - \frac{1}{5} H_3^j(4), \quad (48)$$

where

$$H_3^i(4) = (1 + \delta)F_3^i(4) - \delta^2 F_2^i(4).$$

Thus, equations (42) and (48) fully characterize step $n = 3$ equilibrium deliveries and contracting positions, respectively, as a function of previously taken positions (i.e., $F_3^i(4)$ and $F_2^j(4)$).

5.3.4 The induction step

Based on the results of the three steps above, we now formulate the induction hypothesis.

Definition 1 (*Induction hypothesis*) Consider step $2 \leq n \leq T$, and let $k = 0, \dots, n - 2$. Assume that for all $n - k$, the following structure for SPE deliveries and contract positions holds:

$$q_{n-k}^i = \left[\frac{a}{3} \sum_{h=0}^{n-k-2} (\delta^h - (n-k-1)\delta^{n-k-1}) + \delta^{n-k-1} s_{n-k}^i + \frac{2}{3} H_{n-k}^i(n-k) - \frac{1}{3} H_{n-k}^j(n-k) \right] \frac{1}{\sum_{h=0}^{n-k-1} \delta^h} \quad (49)$$

$$H_{n-k}^i(n) = \frac{k+1}{3+2(k+1)} a \sum_{h=0}^{n-k-2} (\delta^h - (n-k-1)\delta^{n-k-1}) + \frac{3+k+1}{3+2(k+1)} H_{n-k}^i(n+1) - \frac{k+1}{3+2(k+1)} H_{n-k}^j(n+1) \quad (50)$$

$$H_{n-k}^i(n+1) = \sum_{h=0}^{n-k-2} \delta^h F_{n-k}^i(n+1) - \delta^{n-1} \sum_{h=1}^{n-k-2} F_{n-k-h}^i(n+1). \quad (51)$$

It is easy to verify the hypothesis holds for the three-period solution derived above: set $n = 3$ and $k = 0$ for the third step ($n = 3$), $n = 3$ and $k = 1$ for the second step ($n = 2$), and $n = 3$ and $k = 2$ for the last step ($n = 1$). Note that the auxiliary variable k is needed because the induction hypothesis at some n depends on the number of steps following n . Since it greatly simplifies the exposition, we proceed by assuming that the induction hypothesis holds for $n - 1$ and then prove that it holds for n . We thus need to verify that if for step $n - 1$ both spot and forward best responses follow the structure (49)-(51) then they also follow it for step n .

For the spot best responses we need the following result:

Lemma 7 *If (49)-(51) hold for $n - 1$, then*

$$\frac{\partial V_{n-1}^i(s_{n-1}^i, s_{n-1}^j, \mathbf{F}_{n-1}^i(n), \mathbf{F}_{n-1}^j(n))}{\partial s_{n-1}^i} = \frac{\delta^{n-2}}{\sum_{h=0}^{n-2} \delta^h} ((n-1)a - 2s_{n-1}^i - s_{n-1}^j + \sum_{h=1}^{n-2} F_{n-h}^i(n)). \quad (52)$$

Proof. The payoff V_{n-1}^i amounts to revenues from the remaining periods in equilibrium, starting from step $n - 1$. It is easier to keep track of these revenues forward in time than backward through steps. Therefore, we make a temporary notational shift, and let 1 denote step $n - 1$ and M the last step. Thus,

$$V_{n-1}^i \equiv V_1^i = \sum_{h=1}^M \delta^{h-1} p_h^s \cdot (q_h^i - F_h^i(n)) + C$$

where C is again some constant that captures open contract positions. We have

$$\frac{\partial V_1^i(\cdot)}{\partial s_1^i} = \sum_{h=1}^M \delta^{h-1} (a - 2q_h^i - q_h^j + F_h^i(n)) \frac{\partial q_h^i}{\partial s_h^i} \frac{\partial s_h^i}{\partial s_1^i}. \quad (53)$$

Take the delivery rule (49) and set $M = n$ and $k = 0$ to obtain

$$\frac{\partial q_1^i}{\partial s_1^i} \frac{\partial s_1^i}{\partial s_1^i} = \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} = \frac{\delta^{M-1}(1-\delta)}{1-\delta^M}.$$

Similarly, set $M = n$ and $k = 1$ to obtain

$$\frac{\partial q_2^i}{\partial s_2^i} = \frac{\delta^{M-2}}{\sum_{h=0}^{M-2} \delta^h} = \frac{\delta^{M-2}(1-\delta)}{1-\delta^{M-1}}$$

and

$$\frac{\partial q_2^i}{\partial s_2^i} \frac{\partial s_2^i}{\partial s_1^i} = \frac{\partial q_2^i}{\partial s_2^i} \left(1 - \frac{\partial q_1^i}{\partial s_1^i}\right) = \frac{\delta^{M-2}(1-\delta)}{1-\delta^{M-1}} \left(1 - \frac{\delta^{M-1}(1-\delta)}{1-\delta^M}\right) = \frac{\delta^{M-2}(1-\delta)}{1-\delta^M}.$$

And finally, set $M = n$ and $k = 2$ to obtain

$$\frac{\partial q_3^i}{\partial s_3^i} = \frac{\delta^{M-3}}{\sum_{h=0}^{M-3} \delta^h} = \frac{\delta^{M-3}(1-\delta)}{1-\delta^{M-2}}$$

and

$$\frac{\partial q_3^i}{\partial s_3^i} \frac{\partial s_3^i}{\partial s_1^i} = \frac{\partial q_3^i}{\partial s_3^i} \left(1 - \frac{\partial q_1^i}{\partial s_1^i} - \frac{\partial q_2^i}{\partial s_2^i} \frac{\partial s_2^i}{\partial s_1^i}\right) = \frac{\delta^{M-3}(1-\delta)}{1-\delta^M}.$$

From this exercise we can see that

$$\frac{\partial q_l^i}{\partial s_l^i} \frac{\partial s_l^i}{\partial s_1^i} \delta^{l-1} = \frac{\delta^{M-l}(1-\delta)}{1-\delta^M} \delta^{l-1} = \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} \quad (54)$$

for all l . Replacing (54) into (53) yields

$$\begin{aligned} \frac{\partial V_1^i(\cdot)}{\partial s_1^i} &= \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} \sum_{h=1}^M (a - 2q_h^i - q_h^j + F_h^i(n)) \\ &= \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} (Ma - 2 \sum_{h=1}^M q_h^i - \sum_{h=1}^M q_h^j + \sum_{h=1}^{M-1} F_h^i(n)) \\ &= \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} (Ma - 2s_1^i - s_1^j + \sum_{h=1}^{M-1} F_h^i(n)) \end{aligned}$$

where the last line follows from the stock exhaustion conditions. Note that we add the contracts up to $M - 1$ since we can disregard contracting for the last period. Switching back to the "step" notation, where there are $n - 1$ steps rather than M periods, gives (52). ■

Analogously, for the forward market best responses we need the following result:

Lemma 8 *If (49)-(51) hold for $n - 1$, then*

$$\frac{\partial V_{n-1}^i(s_{n-1}^i, s_{n-1}^j, \mathbf{F}_{n-1}^i(n), \mathbf{F}_{n-1}^j(n))}{\partial s_{n-1}^j} = \frac{\delta^{n-2}}{\sum_{h=0}^{n-2} \delta^h} (\sum_{h=1}^{n-2} F_{n-h}^i(n) - s_{n-1}^i). \quad (55)$$

Proof. We make a similar temporary notational shift as above and write

$$V_1^i = \sum_{h=1}^M \delta^{h-1} p_h^s \cdot (q_h^i - F_h^i(n)) + C$$

from which we obtain

$$\frac{\partial V_1^i(\cdot)}{\partial s_1^j} = \sum_{h=1}^M \delta^{h-1} (F_h^i - q_h^i) \frac{\partial q_h^j}{\partial s_h^j} \frac{\partial s_h^j}{\partial s_1^j}.$$

But from Lemma 7 we know that

$$\delta^{l-1} \frac{\partial q_l^j}{\partial s_l^j} \frac{\partial s_l^j}{\partial s_1^j} = \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h},$$

hence

$$\frac{\partial V_1^i(\cdot)}{\partial s_1^j} = \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} \sum_{h=1}^M (F_h^i(n) - q_h^i) = \frac{\delta^{M-1}}{\sum_{h=0}^{M-1} \delta^h} \left(\sum_{h=1}^{M-1} F_h^i(n) - s_1^i \right)$$

where the term after the second equality follows since there is no contracting for the last period and sales add up to the stock. Switching back to the "step" notation gives (55). \blacksquare

Assuming now the hypothesis holds for $n - 1$, consider the spot market best responses at n . The latter must satisfy

$$\frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n))]}{\partial q_n^i} + \delta \frac{\partial V_{n-1}^i(s_{n-1}^i, s_{n-1}^j, \mathbf{F}_{n-1}^i(n), \mathbf{F}_{n-1}^j(n))}{\partial s_{n-1}^i} \frac{\partial s_{n-1}^i}{\partial q_n^i} = 0.$$

for both i and j . But from Lemma 7 the above reduces to

$$a - 2q_n^i - q_n^j + F_n^i(n) - \delta \frac{\delta^{n-2}}{\sum_{h=0}^{n-2} \delta^h} ((n-1)a - 2s_{n-1}^i - s_{n-1}^j + \sum_{h=1}^{n-2} F_{n-h}^i(n)) = 0.$$

Rearranging gives

$$\sum_{h=0}^{n-2} \delta^h (a - 2q_n^i - q_n^j + F_n^i(n)) = \delta^{n-1} [(n-1)a - 2(s_n^i - q_n^i) - (s_n^j - q_n^j) + \sum_{h=1}^{n-2} F_{n-h}^i(n)],$$

therefore

$$\begin{aligned} 2q_n^i \sum_{h=0}^{n-1} \delta^h &= -q_n^j \sum_{h=0}^{n-1} \delta^h + 2\delta^{n-1} s_n^i + \delta^{n-1} s_n^j + a \left(\sum_{h=0}^{n-2} \delta^h - (n-1)\delta^{n-1} \right) \\ &\quad + \sum_{h=0}^{n-2} \delta^h F_n^i(n) - \delta^{n-1} \sum_{h=1}^{n-2} F_{n-h}^i(n). \end{aligned}$$

Use this equation for both i and j to obtain the spot best responses

$$q_n^i = \left[\frac{a}{3} \sum_{h=0}^{n-2} (\delta^h - (n-1)\delta^{n-1}) + \delta^{n-1} s_n^i + \frac{2}{3} H_n^i(n) - \frac{1}{3} H_n^j(n) \right] \frac{1}{\sum_{h=0}^{n-1} \delta^h} \quad (56)$$

for both i and j and where

$$H_n^i(n) = \sum_{h=0}^{n-2} \delta^h F_n^i(n) - \delta^{n-1} \sum_{h=1}^{n-2} F_{n-h}^i(n).$$

Consider next the forward best responses at the forward stage of step n . The choice $F_n^i(n)$ for each firm i satisfies

$$\begin{aligned} & \left[\left\{ \frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n+1))]}{\partial q_n^i} + \delta \frac{\partial V_{n-1}^i}{\partial s_{n-1}^i} \frac{\partial s_{n-1}^i}{\partial q_n^i} \right\} \frac{\partial q_n^i}{\partial H_n^i(n)} \right. \\ & \left. + \left\{ \frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n+1))]}{\partial q_n^j} + \delta \frac{\partial V_{n-1}^i}{\partial s_{n-1}^j} \frac{\partial s_{n-1}^j}{\partial q_n^j} \right\} \frac{\partial q_n^j}{\partial H_n^i(n)} \right] \frac{\partial H_n^i(n)}{\partial F_n^i(n)} = 0 \end{aligned}$$

where $V_{n-1}^i = V_{n-1}^i(s_{n-1}^i, s_{n-1}^j, \mathbf{F}_{n-1}^i(n+1), \mathbf{F}_{n-1}^j(n+1))$. From Lemmas 7 and 8 we have the changes in the continuation values, and from (56) the effect $\partial q_n^i / \partial H_n^i(n)$; thus, the expression simplifies to (recall that $\partial H_n^i(n) / \partial F_n^i(n) \neq 0$)

$$\begin{aligned} & \left[\left\{ \frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n+1))]}{\partial q_n^i} - \delta \frac{\delta^{n-2}}{\sum_{h=0}^{n-2} \delta^h} ((n-1)a - 2s_{n-1}^i - s_{n-1}^j + \sum_{h=1}^{n-1} F_{n-h}^i(n+1)) \right\} \frac{2}{3} \right. \\ & \left. + \left\{ \frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n+1))]}{\partial q_n^j} - \delta \frac{\delta^{n-2}}{\sum_{h=0}^{n-2} \delta^h} (\sum_{h=1}^{n-2} F_{n-h}^i(n+1) - s_{n-1}^i) \right\} \frac{-1}{3} \right] \frac{\sum_{h=0}^{n-2} \delta^h}{\sum_{h=0}^{n-1} \delta^h} = 0 \end{aligned}$$

Solving for spot quantities of i and j gives

$$q_n^i = \left[\frac{2a}{5} \sum_{h=0}^{n-2} (\delta^h - (n-1)\delta^{n-1}) + \delta^{n-1} s_{n-k}^i + \frac{3}{5} H_n^i(n+1) - \frac{2}{5} H_n^j(n+1) \right] \frac{1}{\sum_{h=0}^{n-1} \delta^h} \quad (57)$$

Equating (56) and (57) for both i and j gives the forward best responses

$$H_n^i(n) = \sum_{h=0}^{n-2} \frac{a}{5} (\delta^h - (n-1)\delta^{n-1}) + \frac{4}{5} H_n^i(n+1) - \frac{1}{5} H_n^j(n+1)$$

where

$$H_n^i(n+1) = \sum_{h=0}^{n-2} \delta^h F_n^i(n+1) - \delta^{n-1} \sum_{h=1}^{n-2} F_{n-h}^i(n+1).$$

We have thus verified the induction hypothesis for n and $k = 0$. Consider now n and $k > 0$. The condition for best responses $F_{n-k}^i(n)$ has two parts. First, the effect on deliveries at n is zero by the first-order condition for $F_n^i(n)$

$$\begin{aligned}
& \left[\left\{ \frac{\partial [p_n^s \cdot (q_n^i - F_n^i(n+1))]}{\partial q_n^i} + \delta \frac{\partial V_{n-1}^i}{\partial s_{n-1}^i} \frac{\partial s_{n-1}^i}{\partial q_n^i} \right\} \frac{\partial q_n^i}{\partial H_n^i(n)} + \right. \\
& \left. + \left\{ \frac{\partial [p_n^s \cdot (q_n^j - F_n^j(n+1))]}{\partial q_n^j} + \delta \frac{\partial V_{n-1}^j}{\partial s_{n-1}^j} \frac{\partial s_{n-1}^j}{\partial q_n^j} \right\} \frac{\partial q_{n-1}^j}{\partial H_n^i(n)} \right] \frac{\partial H_n^i(n)}{\partial F_{n-k}^i(n)} = 0,
\end{aligned}$$

so we are left with the effect on future deliveries

$$\begin{aligned}
& \left[\left\{ \frac{\partial [p_{n-k}^s \cdot (q_{n-k}^i - F_{n-k}^i(n+1))]}{\partial q_{n-k}^i} + \delta \frac{\partial V_{n-k-1}^i}{\partial s_{n-k-1}^i} \frac{\partial s_{n-k-1}^i}{\partial q_{n-k}^i} \right\} \frac{\partial q_{n-k}^i}{\partial H_{n-k}^i(n)} \right. \\
& \left. + \left\{ \frac{\partial [p_{n-k}^s \cdot (q_{n-k}^j - F_{n-k}^j(n+1))]}{\partial q_{n-k}^j} + \delta \frac{\partial V_{n-k-1}^j}{\partial s_{n-k-1}^j} \frac{\partial s_{n-k-1}^j}{\partial q_{n-k}^j} \right\} \frac{\partial q_{n-k}^j}{\partial H_{n-k}^i(n)} \right] \frac{\partial H_{n-k}^i(n)}{\partial F_{n-k}^i(n)} = 0.
\end{aligned}$$

We have already solved this condition for the case $k = 0$. The only substantial difference between cases $k = 0$ and $k > 0$ is that the spot $n - k$ is served k times by the forward markets. Using the procedures outlined above and the functional forms for the derivatives of the value functions for trading round k yields (49), (50) and (51). This completes the induction step.

5.3.5 The equilibrium delivery

We now use the induction hypothesis, shown to hold for all steps, to derive the SPE delivery rule in the text. At $t = 1$ (i.e., $n = T$), there are no contracts from the past, $H_n^i(T+1) = 0$. This, together with (49)-(51) and the symmetry of the positions, implies

$$\begin{aligned}
q_{n-k}^i &= \left[\frac{a}{3} \sum_{h=0}^{n-k-2} (\delta^h - (n-k-1)\delta^{n-k-1}) + \delta^{n-k-1} s_{n-k}^i \right. \\
& \left. + \frac{k+1}{3+2(k+1)} a \sum_{h=0}^{n-k-2} (\delta^h - (n-k-1)\delta^{n-k-1}) \right] \frac{1}{\sum_{h=0}^{n-k-1} \delta^h}.
\end{aligned}$$

It is matter of direct verification that setting $n = T$ and $k = 0$ leads to the SPE delivery for $t = 1$ in Proposition 3, $n = T$ and $k = 1$ for $t = 2$, and so on.

5.4 Proof of Proposition 4

Let τ denote the time that has elapsed after t periods, so if $\Delta > 0$ is the period length, then $t = \tau/\Delta$. Likewise, let $\Upsilon(\Delta)$ denote the time it takes firms to exhaust their stocks in equilibrium for some period length Δ , so $\Upsilon(\Delta)/\Delta = T(\Delta) < N(\Delta)$, where $T(\Delta)$ is the period of exhaustion and $N(\Delta)$ is the number of periods of the game. Thus, step n

of Proposition 3 (i.e., number of steps or periods before reaching the exhaustion period $T(\Delta)$) can be now expressed as

$$n = \frac{\Upsilon(\Delta) - \tau}{\Delta} + 1. \quad (58)$$

Replacing (58), the continuous-time discount factor $\delta = e^{-r\Delta}$ (r is the instantaneous interest rate) and $t = \tau/\Delta$ into (18) we obtain (recall that s_t^i must be scaled by the period length)

$$q_t^i = q_t^j = \left\{ \frac{a}{3} \left[\sum_{h=0}^{\frac{\Upsilon(\Delta)-\tau}{\Delta}-1} e^{-r\Delta h} - \left(\frac{\Upsilon(\Delta) - \tau}{\Delta} \right) e^{-r(\Upsilon(\Delta)-\tau)} \right] \left[1 + \frac{\tau/\Delta}{3 + 2\tau/\Delta} \right] + e^{-r(\Upsilon(\Delta)-\tau)} \frac{s_t^i}{\Delta} \right\} \frac{1}{\sum_{h=0}^{\frac{\Upsilon(\Delta)-\tau}{\Delta}-1} e^{-r\Delta h}}. \quad (59)$$

Deliveries are positive for all Δ , so $\Upsilon(\Delta)$ must converge to some finite value $\Upsilon > 0$; otherwise the resource constraint is not satisfied. Letting $\Delta \rightarrow 0$ we obtain the instantaneous equilibrium delivery

$$q_\tau^i = \frac{a (e^{r(\Upsilon-\tau)} - 1 - r(\Upsilon - \tau))}{2 (e^{r(\Upsilon-\tau)} - 1)} + \frac{r s_\tau^i}{e^{r(\Upsilon-\tau)} - 1}. \quad (60)$$

Consider now the socially optimal (continuous-time) delivery path starting at time τ and given an overall (remaining) stock of $s_\tau = 2s_\tau^i = 2s_\tau^j$. Since prices grow at the rate of interest along the efficient path, at any time $\tau \leq \tau' \leq \Upsilon$ the socially optimal delivery satisfy

$$a - q_{\tau'}^* = (a - q_\tau^*) e^{r(\tau'-\tau)}.$$

And using the exhaustion condition

$$\int_\tau^\Upsilon q_{\tau'}^* d\tau' = s_\tau,$$

yields

$$q_\tau^* = a \frac{(e^{r(\Upsilon-\tau)} - 1 - r(\Upsilon - \tau))}{e^{r(\Upsilon-\tau)} - 1} + \frac{r s_\tau}{e^{r(\Upsilon-\tau)} - 1} \quad (61)$$

which is twice the delivery shown in (60).

References

- [1] Allaz, B., and J.-L. Vila (1993), Cournot competition, forward markets and efficiency, *Journal of Economic Theory* 59, 1-16.

- [2] Ausubel, L. M., and R. J. Deneckere (1989), Reputation and bargaining in durable goods monopoly, *Econometrica* 57, 511-531.
- [3] Bhaskar, V., (2008). Dynamic countervailing power with public and private monitoring, mimeo, UCL.
- [4] Biglaiser, G., and Vettas, N. (2008), Dynamic price competition with capacity constraints and strategic buyers, working paper, University of North Carolina..
- [5] Bushnell, J., E. Mansur and C. Saravia (2008), Vertical arrangements, market structure and competition: An analysis of restructured U.S. electricity markets, *American Economic Review* 98, 237-266.
- [6] Dudey, M. (1992), Dynamic Edgeworth-Bertrand competition, *Quarterly Journal of Economics* 107, 1461-1477.
- [7] Fabra, N., and J. Toro (2005), Price wars and collusion in the Spanish electricity market, *International Journal of Industrial Organization* 23, 155-181.
- [8] Fabra, N., and M.-A. de Frutos (2010), How to award pro-competitive forward contracts: The case of electricity auctions, working paper, Universidad Carlos III de Madrid.
- [9] Gaudet, G., M. Moreaux and S.W. Salant (2002), Private Storage of Common Property, *Journal of Environmental Economics and Management* 43, 280-302.
- [10] Gaudet, G. (2007), Natural resource economics under the rule of Hotelling, *Canadian Journal of Economics* 40, 1034-1059.
- [11] Green, R. (1999), The Electricity Contract Market in England and Wales, *Journal of Industrial Economics* 47, 107-124.
- [12] Gul, F. (1987), Noncooperative collusion in durable goods oligopoly, *RAND Journal of Economics* 18, 248-254.
- [13] Hotelling, H. (1931), The economics of exhaustible resources, *Journal of Political Economy* 39, 137-175.
- [14] Kahn, C. M. (1986), The durable goods monopolist and consistency with increasing costs, *Econometrica* 54, 275-94.

- [15] Lewis, T. and R.Schmalensee (1980), On oligopolistic markets for nonrenewable resources, *Quarterly Journal of Economics* 95, 475-491.
- [16] Liski, M., and J.-P. Montero (2009), Forward trading in exhaustible-resource oligopoly, working paper, CEEPR-MIT.
- [17] Liski, M., and J.-P. Montero (2006), Forward trading and collusion in oligopoly, *Journal of Economic Theory* 131, 212-230.
- [18] Loury, G. (1986), A theory of ‘oil’igopoly: Cournot–Nash equilibrium in exhaustible resources markets with fixed supplies. *International Economic Review* 27, 285–301
- [19] Mahenc, P., and F. Salanié (2004), Softening competition through forward trading, *Journal of Economic Theory* 116, 282-293.
- [20] Phelps, L., and R.M. Harstad (1990), Interaction between resource extraction and futures markets: A game-theoretic analysis, in R. Selten (ed.), *Game Theory in the Behavioral Sciences*, Springer Verlag, Heidelberg.
- [21] Polasky, S. (1992), Do oil producers act as ‘oil’igopolists?, *Journal of Environmental Economics and Management* 23, 216–247.
- [22] Reinganum, J. F., and N.L. Stokey (1985), Oligopoly Extraction of a Common Property Natural Resource: The Importance of the Period of Commitment in Dynamic Games, *International Economic Review* 26, 161-173.
- [23] Salant, S.W. (1976), Exhaustible resources and industrial structure: A Nash-Cournot approach to the world oil market, *Journal of Political Economy* 84, 1079-1093.
- [24] Saloner, G. (1987), Cournot duopoly with two production periods, *Journal of Economic Theory* 42, 183-187.
- [25] Salo, S., and O. Tahvonen (2001), Oligopoly equilibria in nonrenewable resource markets, *Journal of Economic Dynamics and Control*, Volume 25, Issue 5, 671-702.
- [26] Talluri, K., and V. Martinez (2010), Dynamic price competition with fixed capacities, working paper, Universitat Pompeu Fabra.
- [27] Ulph, A., and D. Ulph (1989), Gains and losses from cartelisation in markets for exhaustible resources in the absence of binding future contracts, In *Dynamic Policy*

Games in Economics: Essays in Honor of Piet Verheyen, F. van der Ploeg and A. de Zeeuw (Eds.), North-Holland.

- [28] Wolak, F. (2000), An empirical analysis of the impact of hedge contracts on bidding behavior in competitive electricity markets, *International Economic Journal* 14, 1-39.