Climate policies with non-constant discounting

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Abstract

We consider the climate-policy implications of non-constant time preferences when there is no commitment to future policies. The conceptual and quantitative results follow from the observation that, with time-declining discounting, the unusual delays and persistence of climate impacts provide a commitment device for climate policy-makers, overturning a fundamental climate-policy conclusion from the previous literature that the returns on climate investments should equal the return on capital savings in the economy. We assess the quantitative implications by restricting attention to a parametric class for preferences and technology, and by solving for a time-consistent Markov equilibrium explicitly.

(JEL classification: H43; H41; D61; D91; Q54; E21. Keywords: carbon tax, discounting, climate change, inconsistent preferences)

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1 Introduction

The choice of the long-run discount rate is central when evaluating public projects with very long-run impacts such as the optimal response to climate change. While there is no general consensus on the discount rate to be used for different time horizons, there is certainly very little evidence for using the same constant rate for all horizons. For example, there is recent revealed-preference evidence that “Households discount very long-run cash flows at low rates, assigning high present value to cash flows hundreds of years in the future” (Giglio et al., 2014), consistent with earlier findings based on stated preference surveys. Also, the UK treasury guidelines for policy evaluations recommend that “the discounting of effects in the very long term, the received view is that a lower discount rate for the longer term (beyond 30 years) should be used.”

Yet, for one reason or another, the implications of non-constant discounting for intergenerational cost-benefit analysis have not been expounded. It is not unreasonable to think that policy-makers discount utility gains within their lifetime differently from those after their time. If the next generation policy-makers make the same distinction between short- and long-term discounting, and if their decisions cannot be dictated today, the setting becomes a policy game between agents who make policies in the order they enter the time-line. Then, the question is how to optimally design climate policies when there is no commitment to future policies, as these are under the control of the future decision makers who discount time differently.

We consider the climate-policy implications of non-constant time discounting in such a policy game. Substantially, the point of the paper is to show that the extreme delays and persistence of climate impacts provide a commitment device for policy-makers: the policies of today have a peak impact on future utilities with a considerable delay, that is, after 60-70 years in our quantitative model. Climate policies, when optimally designed, should exploit the commitment to future utility impacts, and, when doing so, they depart from the idea that the same return requirement holds for all investments in the economy. The equilibrium return requirement for climate investments becomes  

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1 For stated-preference evidence, Layton and Brown (2000) and Layton and Levine (2003) used a survey of 376 non-economists, and found a small or no difference in the willingness to pay to prevent future climate change impacts appearing after 60 or 150 years. Weitzman (2001) surveyed 2,160 economists for their best estimate of the appropriate real discount rate to be used for evaluating environmental projects over a long time horizon, and used the data to argue that the policy maker should use a discount rate that declines over time — coming close to zero after 300 years.

2 HM Treasury Green Book 2014, p. 98.
lower than for capital savings: due to the commitment value, climate impacts receive a non-negligible weight over horizons that are significantly longer than those relevant for capital investments in the economy. Intuitively, the climate asset, through its extreme persistence, provides a “golden egg” for present-day policies, with commitment value arising endogenously in the equilibrium.

We assess the commitment value by restricting attention to a parametric class for preferences and technology, and by solving for a time-consistent Markov equilibrium explicitly. The analysis introduces non-constant time preferences in a general-equilibrium growth framework, building on Nordhaus’ approach to climate-economy modelling (2008) and its recent gearing towards the macro traditions by Golosov, Hassler, Krusell, and Tsyvinsky (2014). In addition to the previous literature, we develop and calibrate a tractable representation of the climate-change dynamics to provide a structured quantification the commitment value. The representation allows us to derive a transparent carbon pricing formula, decomposing the contributions of the size, delay, and persistence of climate impacts to the carbon price, and their interaction with the time-structure of preferences. For example, ignoring the delay of impacts — as in Golosov et al. (2014) and the follow-up literature — misses the correct carbon price levels by a factor of two.

Table 1 contains the gist of the quantitative assessment. The model is calibrated to 25 per cent gross savings, when both the short- and long-term annual time discount rate is 2.7 per cent. This is consistent with Nordhaus’ DICE 2007 baseline scenario (Nordhaus, 2007) giving 7.1 Euros per ton of CO$_2$ as the optimal carbon price in the year 2010 (i.e., 34 Dollars per ton C). The first row provides the optimal, consistent-preferences, benchmark carbon price. But, Nordhaus’ number is a benchmark also in another sense: it is the equilibrium carbon tax for a policy-maker who insists on using the economy’s return on capital savings as the return requirement also for climate investments, based on the interpretation that observed capital returns provide an aggregate measure of revealed social preferences. The first row thus remains a benchmark for policy choice, as it dictates the carbon price that is consistent with a climate investment return based on “revealed aggregate social preferences”, even if time preferences are non-constant.

In contrast, the equilibrium Markov policy differentiates between the persistence of

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3 Nordhaus uses an annual pure rate of time preference of 1.5 per cent; our value 2.7 is the equivalent number when adjusting for the difference in the consumption smoothing parameter, and labor productivity growth. See Nordhaus (2008) for detailed documentation of DICE 2007.

4 The approach has authoritative advocates: “[...] As this approach relates to discounting, it requires that we look carefully at the returns of alternative investments —at the real interest rate— as the benchmarks for climate investments.” Nordhaus (2007, p. 692).
capital and climate investments. The persistence gap is important for the planner as each asset has its own commitment value through its effect on future utilities. The Markov policies lead to a prodigious increase in the carbon price, reflecting the policy-maker’s willingness to pay for a commitment to a decrease in long-lasting climate impacts. The Markov equilibrium carbon price is shown on the second row where, for the sake of illustration, we adjust the short- and long-run preference discount rates so that savings remain unaffected, and thus the time preferences revealed by the savings in the economy remain observationally equivalent to those in the first row. For very low long-term discounting, the equilibrium carbon prices ultimately approach those suggested by Stern (2006), presented in the third row, who suggested using the low long-term discount rate for investment decisions over all horizons. 

<table>
<thead>
<tr>
<th>discount rate</th>
<th>short-term</th>
<th>long-term</th>
<th>savings</th>
<th>carbon price</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Nordhaus”</td>
<td>.027</td>
<td>.027</td>
<td>.25</td>
<td>7.1</td>
</tr>
<tr>
<td>Equilibrium</td>
<td>.037</td>
<td>.001</td>
<td>.25</td>
<td>133</td>
</tr>
<tr>
<td>“Stern”</td>
<td>.001</td>
<td>.001</td>
<td>.33</td>
<td>174</td>
</tr>
</tbody>
</table>

Table 1: Carbon prices in EUR/tCO₂ year 2010.

Climate policies based on “revealed aggregate preferences” and those based on Markov strategies are both inefficient. Inefficiency of the Markov policy follows immediately from the gap between investment returns for different assets. The inefficiency under the revealed-preference policy is more subtle. This policy discounts gains from climate investments using the capital returns, but the equilibrium aggregate statistics of the macroeconomy become distorted under non-constant discounting: there is a wedge between the marginal rate of substitution (MRS) and the marginal rate of transformation (MRT). The wedge arises from a shortage of future savings leading to higher capital returns than what the current policy-maker would like to see. As is well-known in cost-benefit analysis, a distorted capital return is not the social rate of return for public investments. For this reason, the equilibrium carbon price, based on the MRS, is higher than Nordhaus’ number, based on the MRT. The Markov policy corrects for the return

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5 Under “Stern” the capital-share of output is fully saved (33 per cent); increasing the capital-share leads to unrealistic savings as discussed, e.g., in Weitzman (2007) and Dasgupta (2008).

6 This distortion is the same as in Barro (1999); and Krusell, Kuruscu, and Smith (2002).

7 See Lind (1982), or, e.g., Dasgupta (2008).
distortions in the economy and, thus, is the optimal policy given the structure of the policy game. Note that such distortions cannot be avoided with time-changing discount rates since policy decisions are de facto made in the order of appearance of policy-makers in the time line.

In the Markov equilibrium, the current and all future policy makers internalize all climate impacts of emissions; distortions arise from the lack of commitment to future savings and emissions. Todays’ savings and climate investments can be strategic substitutes or complements for future savings and climate investments and, thus, lower or above those implemented in the case of full commitment to future actions. For a widely used parametric class of preferences and technologies, covering for example those in Golosov et al. (2014), we show that climate investments are not used for manipulating future savings and vice versa; the “over-investment” in the climate asset reflects purely the greater persistence of the utility impacts in comparison with shorter term capital savings. Thus, for the parametric class considered, the generations “agree” that a lower rate of return should be used for climate investments, so that current climate investments are not undermined by reduced future actions, even though, in principle, such a response is available to agents in equilibrium.

At first glance, our finding that the optimal equilibrium policy is more ambitious than the benchmark “revealed-preference” climate policy may seem surprising given that the actual observed climate policies fall short of the benchmark. In our analysis the focus is on a global planning problem with intergenerational distortions but without more immediate obstacles to policies such as those arising from international free-riding. Our planning problem can help to find rules or institutions that support consistency in public policy-making over time. The rule that insists on using the capital returns for climate investments is a natural benchmark rule to consider; however, the planner can choose, in the Markov equilibrium, a better rule which involves using differential returns between private capital and public climate investments. The analysis thus suggests that the gap between observed, low or non-existent, carbon prices and the optimal one is even larger as the gap that is already suggested by models based on time-consistent preferences.

A Markov policy is a step towards a normative policy that would maximize a multi-

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8 Although the commitment problem is similar to that in Laibson (1997), self-control at the individual level is not the interpretation of the “behavioral bias” in our economy; we think of decision makers as generations as in Phelps and Pollak (1968). In this setting, the appropriate interpretation of hyperbolic discounting is that each generation has a social welfare function that expresses altruism towards long-term beneficiaries (see also Saez-Marti and Weibull, 2005).
generation welfare criterion. While inconsistencies at the level of individuals lead to well-known problems in identifying the welfare-relevant preferences (see, e.g., Chetty, 2015), in our case the welfare implications are straightforward: agents in different periods are distinct and, legitimately, a standard multi-agent Pareto criterion can be applied. In this setting, the Markov outcome remains suboptimal; intuitively, future agents do not take into consideration the effect of their policy on current welfare. But such a link and an improvement in welfare for all can be obtained by constructing a self-enforcing dependence between generations’ policy rules. We call these policies “sustainable”. They are savings and climate policy prescriptions that (i) apply symmetrically to all generations; (ii) are mutually agreeable so that the prescription coincides with what each generation would like to propose for itself and the future; and (iii) are self-enforcing. The Markov policy serves as the default in case no other policy prescription exists that satisfies the conditions, and as the default to which the economy moves back in case that some generation deviates from the sustainable policy rule. A mechanism to support sustainable policies can be thought of as a type of “self-reputation” (Benabou and Tirole 2004), an analogy to individual-level inconsistencies. The emerging contract is unique and leads to coordinated savings and climate policies, bringing the returns closer to each other. Therefore, the self-enforcing sustainable climate policy is almost identical with one that uses a long-term discount rate for both the (coordinated) capital returns and for pricing carbon emissions. An alternative interpretation is that our sustainable policies characterize Stern’s proposed low discount rates with “high” savings and carbon prices as an equilibrium outcome, without assuming such low discounting for all time horizons.

The relevance of hyperbolic discounting in the climate policy analysis has been acknowledged before; however, the broader equilibrium implications have been overlooked. Mastrandrea and Schneider (2001) and Guo, Hepburn, Tol, and Anthoff (2006) include hyperbolic discounting in simulation models assuming that the current decision-makers can choose also the future policies. That is, these papers do not analyze if the policies targeted towards the long-term preferences can be sustained in equilibrium; we introduce such policies in a well-defined sense⁹ Karp (2005), Fujii and Karp (2008) and Karp and Tsur (2011) consider Markov equilibrium climate policies under hyperbolic discounting without commitment to future actions, but these studies employ a stylized setting without intertemporal consumption choices. Our tractable general-equilibrium model

⁹Iverson (December 2012), subsequent to our working paper (June 2012), shows that the Markov equilibrium policy identified in our paper is unique when the equilibrium is constructed as a finite-horizon limit.
features a joint inclusion of macro and climate policy decisions, with hyperbolic time preferences, and a detailed carbon cycle description — these features are all essential for a credible quantitative assessment of the commitment value, as well as for identifying the sustainable prescriptive plans.

We take no stand on what the time-structure of preferences should be. However, several recent conceptual arguments justify the deviation from geometric discounting. First, if we accept that the difficulty of distinguishing long-run outcomes describes well the climate-policy decision problem, then such lack of a precise long-term view can imply a lower long-term discount rate than that for the short-term decisions; see Rubinstein (2003) for the procedural argument. Second, climate investments are public decisions requiring aggregation over heterogenous individual time-preferences, leading again to a non-stationary aggregate time-preference pattern, typically declining with the length of the horizon, for the group of agents considered (Gollier and Zeckhauser, 2005; Jackson and Yariv, 2014). We may also interpret Weitzman’s (2001) study based on the survey of experts’ opinions on discount rates as an aggregation of persistent views. Third, the long-term valuations must by definition look beyond the welfare of the immediate next generation; any pure altruism expressed towards the long-term beneficiaries implies changing utility-weighting over time (Phelps and Pollak 1968 & Saez-Marti and Weibull 2005).

The paper is organized as follows. The next section introduces the infinite-horizon climate-economy model, and Section 3 develops the climate system representation. Section 4 proceeds to the equilibrium analysis and presents the main conceptual results. Section 5 provides the quantitative assessment of the conceptual results. To obtain sharp results in a field dominated by simulation models, we make specific functional assumptions. Section 6 discusses those assumptions, and some robustness analysis as well as extensions to uncertainty and learning. Section 7 concludes. All proofs, unless helpful in the text, are in the Appendix. The supplementary material cited in the text is available in a public folder.

10 From the current perspective, generations living after 400 or, alternatively, after 450 years look the same. That being the case, no additional discounting arises from the added 50 years, while the same time delay commands large discounting in the near term.

11 Follow the link https://www.dropbox.com/sh/q9y9l12j311ac6h/dgYpKVoCMg
2 An infinite horizon climate-economy model

2.1 Technologies and preferences: general setting

Consider a sequence of periods \( t \in \{1, 2, 3, \ldots \} \). The economy’s production possibilities, captured by function \( f_t(k_t, l_t, z_t, s_t) \), depend on capital \( k_t \), labour \( l_t \), current fossil-fuel use \( z_t \), and the emission history (i.e., past fossil-fuel use),

\[
s_t = (z_1, z_2, \ldots, z_{t-2}, z_{t-1}).
\]

History \( s_t \) enters in production for two reasons. First, climate-change that follows from historical emissions changes production possibilities, as usual in climate-economy models. Second, the current fuel use is linked to historical fuel use through energy resources whose availability and the cost of use depends on the past usage. For the parametric class that we detail below, we abstract from the latter type of history dependence since the scarcity of conventional fossil-fuel resources is not binding when the climate policies are in place (see also Golosov et al., 2014). The economy has one final good. The closed-form solutions require that capital depreciates in one period, leading to the following budget accounting equation between period \( t \) and \( t + 1 \):

\[
c_t + k_{t+1} = y_t = f_t(k_t, l_t, z_t, s_t),
\]

where \( c_t \) is total consumption, \( k_{t+1} \) is capital built for the next period, and \( y_t \) is gross output. The planner representing consumers at time \( t \) makes the consumption, fuel use, labor allocation, and investment decisions. This period and future period planners’ choices generate sequence \( \{c_\tau, z_\tau, k_\tau\}_{\tau=t}^{\infty} \) and per-period utilities, denoted by \( u_\tau \), whose discounted sum defines the welfare of generation \( t \) as

\[
w_t = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau
\]

where discounting is geometric and defined by factor \( 0 < \delta < 1 \) for all dates excluding the current date when \( \beta \neq 1 \). We interpret planners \( t = 1, 2, 3, \ldots \) as distinct agents, discounting their lifetime postponement of utility gains with factor \( \beta \delta \) and then later postponements, after their life, with \( \delta \). While this is the standard quasi-hyperbolic discount function formulation as, for example, in Krusell et al. (2002), there is no individual-level behavioral inconsistency but, rather, only a differential treatment of own and others’ utilities in discounting (as in Phelps and Pollak, 1968). In fact, the parametric model below remains tractable for an arbitrary sequence of discount factors, under certain conditions for boundedness, but the quasi-hyperbolic approximation allows sharper analytical
results.\textsuperscript{12} We take the discount function as a primitive element but, equivalently, one can take altruistic weights on future generations’ welfares as the primitive element and construct a discount function for utilities.\textsuperscript{13}

\section{2.2 The economy}

We build on Brock-Mirman (1972) for the climate-economy interactions, following Golosov et al. (2014), but our approaches to the climate dynamics, preferences, and equilibrium interactions are substantially different. We pull together the production structure as follows:

\begin{align}
  y_t &= k_t^\alpha A_t(l_{y,t}, e_t)\omega(s_t) \\
  e_t &= E_t(z_{t},l_{e,t}) \\
  l_{y,t} + l_{e,t} &= l_t \\
  \omega(s_t) &= \exp(-D_t), \\
  D_t &= \sum_{\tau=1}^{\infty} \theta_{\tau} z_{t-\tau}
\end{align}

Gross production consists of: (i) the Cobb-Douglas capital contribution $k_t^\alpha$ with $0 < \alpha < 1$; (ii) function $A_t(l_{y,t}, e_t)$ for the energy-labour composite in the final-good production with $l_{y,t}$ denoting labor input and $e_t$ total energy use in the economy; (iii) total energy $e_t = E_t(z_{t},l_{e,t})$ using fossil fuels $z_{t}$ and labour $l_{e,t}$; and (iv) the climate impact given by function $\omega(s_t)$ capturing the output loss of production depending on the history of emissions from fossil-fuel use. This parametric class for production together with preferences defined below allow a Markov structure for policies, and thus the determination of the currently optimal policies depending only on the state of the economy, say, at year 2010. In this sense, the currently optimal policies become free of the details of the energy sector as captured by $A_t$ and $E_t$, although the future development of the economy and thus future policies depend on these details.\textsuperscript{14} Here, we merely assume

\textsuperscript{12}Moreover, given the limited data on longer-term discount functions, the advantages of more flexible discount functions are not clear. See Iverson (2014) for the extension of our analysis to the flexible discounting case.

\textsuperscript{13}We explicate this in the Appendix with a three-period model; see Saez-Marti and Weibull (2005) for the general equivalence between generation-specific welfare functionals and discount functions. Note that the preferences are specific for generation $t$, and in that sense, $w_t$ is different from the generation-independent social welfare function (SWF) as discussed, e.g., in Goulder and Williams (2012) and Kaplow et al. (2010).

\textsuperscript{14}We study the future scenarios and specify $A_t$ and $E_t$ in detail in our longer working paper Ger-
that the final-good and energy-sector outputs are differentiable, increasing, and strictly
conceave in labor, energy, and carbon inputs.

The utility function is logarithmic in consumption and, through a separable linear
term, we also include the possibility of intangible damages associated with climate change:

\[ u_t = \ln\left(\frac{c_t}{l_t}\right) - \Delta_u D_t. \]  

(8)

where \( \Delta_u \geq 0 \) is a given parameter. We include \( \Delta_u D_t \) to allow a flexible interpretation
of climate impacts; we develop an aggregate measure of welfare losses covering both
direct utility and output losses. In the calibration, we let \( \Delta_u = 0 \) to maintain an easy
comparison with the previous studies.\textsuperscript{15, 16}

\section{Damages and carbon cycle}

Equations (6)-(7) show that climate damages are interpreted as reduced output, and they
depend on the history of emissions through the state variable \( D_t \) that measures the global
mean temperature increase. The weight structure of past emissions in (7) is derived from
a Markov diffusion process of carbon between various carbon reservoirs in the atmosphere,
oceans and biosphere (see Maier-Reimer and Hasselman 1987). Emissions \( z_t \) enter the
atmospheric \( CO_2 \) reservoir, and slowly diffuse to the other reservoirs. The deep ocean
is the largest reservoir, and the major sink of atmospheric \( CO_2 \). We calibrate this reservoir
system, and, in the analysis below, by a linear transformation obtain an isomorphic
lagh\&Liski (2012). Emissions can decline through energy savings, obtained by substituting labor \( l_{y,t} \)
for total energy \( e_t \). Emissions can also decline through “de-carbonization” that involves substituting
non-carbon inputs for carbon energy inputs \( z_t \) in energy production; de-carbonization is obtained by al-
locating the total energy labor \( l_{e,t} \) further between carbon and non-carbon energy sectors. Typically, the
climate-economy adjustment paths feature early emissions reductions through energy savings, whereas
de-carbonization is necessary for achieving long-term reduction targets. See Gerlagh\&Liski (2012).

15See Tol (2009) for a review of the existing damage estimates; the estimates for intangible damages
are sometimes included, but they are very uncertain and mostly missing. The carbon pricing formulas
help to transform output losses into equivalent intangible losses to gauge the relative magnitudes of such
losses that can be associated with a given carbon price level.

16Note that we consider average utility in our analysis. Alternatively, we can write aggregate utility
within a period by multiplying utility with population size, \( u_t = l_t \ln\left(c_t/l_t\right) - l_t \Delta_u D_t \). The latter approach
is feasible but it leads to considerable complications in the formulas below. Scaling the objective with
labor rules out stationary strategies — they become dependent on future population dynamics —, and
also impedes a clear interpretation of inconsistencies in discounting. While the formulas in the Lemmas
depend on the use of an average utility variable, the substance of the Propositions is not altered. The
expressions for this case are available on request
decoupled system of “atmospheric boxes” where the diffusion pattern between the boxes is eliminated. The reservoirs contain physical carbon stocks measured in Teratons of carbon dioxide \([TtCO_2]\). These quantities are denoted by a \(n \times 1\) vector \(L_t = (L_{1,t}, ..., L_{n,t})\). In each period, share \(b_j\) of total emissions \(z_t\) enters reservoir \(j\), and the shares sum to 1. The diffusion between the reservoirs is described through a \(n \times n\) matrix \(M\) that has real and distinct eigenvalues \(\lambda_1, ..., \lambda_n\). Dynamics satisfy

\[
L_{t+1} = ML_t + bz_t. \tag{9}
\]

**Definition 1** (*closed carbon cycle*) No \(CO_2\) leaves the system: column elements of \(M\) sum to one.

Using the eigen-decomposition theorem of linear algebra, we can define the linear transformation of co-ordinates \(H_t = Q^{-1}L_t\) where \(Q = [v_1 \ldots v_n]\) is a matrix of linearly independent eigenvectors \(v_\lambda\) such that

\[
Q^{-1}MQ = \Lambda = \text{diag}[\lambda_1, ..., \lambda_n].
\]

We obtain

\[
H_{t+1} = Q^{-1}L_{t+1} = Q^{-1}MQH_t + Q^{-1}bz_t
= \Lambda H_t + Q^{-1}bz_t,
\]

which enables us to write the (uncoupled) dynamics of the vector \(H_t\) as

\[
H_{i,t+1} = \lambda_i H_{i,t} + c_i z_t
\]

where \(\lambda_i\) are the eigenvalues, and \(c = Q^{-1}b\). This defines the vector of climate units (“boxes”) \(H_t\) that have independent dynamics but that can be reverted back to \(L_t\) to obtain the original physical interpretation.

For the calibration, we consider only three climate reservoirs: atmosphere and upper ocean reservoir \((L_{1,t})\), biomass \((L_{2,t})\), and deep oceans \((L_{3,t})\). For the greenhouse effect, we are interested in the total atmospheric \(CO_2\) stock. Reservoir \(L_{1,t}\) contains both atmosphere and upper ocean carbon that almost perfectly mix within a ten-year period; we can find the atmospheric stock by correcting for the amount that is stored in the upper oceans. Let \(\mu\) be the factor that corrects for the \(CO_2\) stored in the upper ocean reservoir, so that the total atmospheric \(CO_2\) stock is

\[
S_t = \frac{L_{1,t}}{1 + \mu}.
\]
Let $q_{1,i}$ denote the first row of $Q_i$, corresponding to reservoir $L_{1,t}$. Then, the development of the atmospheric $CO_2$ in terms of the climate boxes is

$$S_t = \sum_i q_{1,i}H_{i,t} \frac{1}{1+\mu}.$$  

This allows the following breakdown: $S_{i,t} = \frac{q_{1,i}}{1+\mu}H_{i,t}$, $a = \frac{q_{1,i}}{1+\mu}Q^{-1}b$, $\eta_i = 1 - \lambda_i$, and

$$S_{i,t+1} = (1 - \eta_i)S_{i,t} + a_i z_t$$  

$$S_t = \sum_{i \in \mathcal{I}} S_{i,t}.$$ 

This is now a system of atmospheric carbon stocks where depreciation factors are defined by eigenvalues from the original physical representation. When no carbon can leave the system, we know one eigenvalue, $\lambda_i = 1$.

**Remark 1** For a closed carbon cycle, one box $i \in \mathcal{I}$ has no depreciation, $\eta_i = 0$.

This observation will have important economic implications when the discount rate is small. We say that the carbon cycle has incomplete absorption if this box is non-negligible:

**Definition 2** (incomplete absorption) Some $CO_2$ remains forever in the atmosphere: there is one box $i \in \mathcal{I}$ that has no depreciation, $\eta_i = 0$ and is non-negligible, $a_i > 0$.

The carbon cycle description is well-rooted in natural science; however, the dependence of temperatures on carbon concentrations and the resulting damages are more speculative. Following Hooss et al (2001, table 2), assume a steady-state relationship between temperatures, $T$, and steady-state concentrations $T = \varphi(S)$. Typically, the assumed relationship is concave, for example, logarithmic. Damages, in turn, are a function of the temperature $D_t = \psi(T_t)$ where $\psi(T)$ is convex. The composition of a convex damage and concave climate sensitivity is approximated by a linear function:

$$\psi'(\varphi(S_t))\varphi'(S_t) \approx \pi$$

---

17 Note also that if the model is run in almost continuous time, that is, with short periods so that most of the emissions enter the atmosphere, $b_1 = 1$, it follows that $\sum_i a_i = 1/(1+\mu)$. Otherwise, we have $\sum_i a_i < 1/(1+\mu)$.

18 See Pindyck (2013) for a critical review.

19 Indeed, the early calculations by Nordhaus (1991) based on local linearization, are surprisingly close to later calculations based on his DICE model with a fully-fledged carbon-cycle temperature module, apart from changes in parameter values based on new insights from the natural science literature.
with $\pi > 0$, a constant characterizing sensitivity of damages to the atmospheric $CO_2$.

Let $\varepsilon$ be the adjustment speed of temperatures and damages, so that we can write for the dynamics of damages:

$$D_t = D_{t-1} + \varepsilon(\pi S_t - D_{t-1}).$$

(12)

This representation of carbon cycle and damages leads to the following analytical emissions-damage response.

**Theorem 1** For the multi-reservoir model with linear damage sensitivity (9)-(12), the time-path of the damage response following emissions at time $t$ is

$$\frac{dD_{t+\tau}}{dz_t} = \theta_\tau = \sum_{i \in I} a_i\pi\varepsilon \frac{(1 - \eta_i)^\tau - (1 - \varepsilon)^\tau}{\varepsilon - \eta_i} > 0,$$

where

$$\eta_i = 1 - \lambda_i$$

$$a_i = \frac{q_{1,i} - c_i}{1 + \mu}$$

For a one-box model (with no indexes $i$), the maximum impact occurs at time between the temperature lifetime $1/\varepsilon$ and the atmospheric $CO_2$ lifetime $1/\eta$.\textsuperscript{22}

Theorem 1 describes the carbon cycle in terms of a system of independent atmospheric boxes, where $I$ denotes the set of boxes, with share $0 < a_i < 0$ of annual emissions entering box $i \in I$, and $\eta_i < 1$ its carbon depreciation factor. The essence of the response is very intuitive. Parameter $\eta_i$ captures, for example, the carbon uptake from the atmosphere by forests and other biomass, and oceans. The term $(1 - \eta_i)^\tau$ measures how much of carbon $z_t$ still lives in box $i$, and the term $-(1 - \varepsilon)^\tau$ captures the slow temperature adjustment in the earth system. The limiting cases are revealing. Consider one $CO_2$ box, so that the share parameter is $a = 1$. If atmospheric carbon-dioxide does not depreciate at all, $\eta = 0$, then the temperature slowly converges at speed $\varepsilon$ to the long-run equilibrium damage sensitivity $\pi$, giving $\theta_\tau = \pi[1 - (1 - \varepsilon)^\tau]$. If atmospheric carbon-dioxide depreciates fully, $\eta = 1$, the temperature immediately adjusts to $\pi\varepsilon$.

\textsuperscript{20}Section 6 reports our sensitivity analysis of the results to this approximation.

\textsuperscript{21}The equation follows from an explicit gradual temperature adjustment process, as modeled in DICE also. See Gerlagh and Liski (2012) for details.

\textsuperscript{22}The $CO_2$ lifetime is the expected number of periods that an emitted CO2 particle remains in the atmosphere. The temperature life time is the average duration that a fictitious temperature shock persists.
and then slowly converges to zero, \( \theta_{\tau} = \pi \varepsilon (1 - \varepsilon)^{\tau - 1} \). If temperature adjustment is immediate, \( \varepsilon = 1 \), then the temperature response function directly follows the carbon-dioxide depreciation \( \theta_{\tau} = \pi (1 - \eta)^{\tau - 1} \). If temperature adjustment is absent, \( \varepsilon = 0 \), there is no response, \( \theta_{\tau} = 0 \).

Figure 1 shows the life-path of losses (percentage of total output) caused by an impulse of one Teraton of Carbon \([\text{TtCO}_2]\) in the first period, contrasted with a counterfactual path without the carbon impulse.\(^{23}\) The output loss is thus measured per \( \text{TtCO}_2 \), and it equals \( 1 - \exp(-\theta_{\tau}) \), \( \tau \) periods after the impulse. The graphs are obtained by calibrating this damage-response, that is, weights \( (\theta_{\tau})_{\tau \geq 1} \) in (7), to three cases.\(^{24}\) Matching Golosov et al.’s (2014) specification produces an immediate damage peak and a fat tail of impacts, while calibrating to the DICE model shows an emissions-damage peak after 60 years with a thinner tail. Our model, that we calibrate with data from the natural sciences literature, produces a combination of the effects: a peak in the emission-damage response function after about 60 years and a fat tail; about 16 per cent of emissions do not depreciate within the horizon of thousand years.

---

\(^{23}\)One \( \text{TtCO}_2 \) equals about 25 years of global \( \text{CO}_2 \) emissions at current levels (40 \( \text{GtCO}_2/\text{yr.} \)).

\(^{24}\)See Appendix for the details of the experiment.
Our emissions-damage response, used in the quantitative part and depicted in Figure 1 (“this paper”) has three boxes calibrated as follows. The physical data on carbon emissions, stocks in various reservoirs, and the observed concentration developments are used to calibrate a three-box carbon cycle representation leading to the following emission shares and depreciation factors per decade:

\[
\begin{align*}
a &= (0.163, 0.184, 0.449) \\
\eta &= (0.074, 0.470)
\end{align*}
\]

Thus, about 16 per cent of carbon emissions does not depreciate while about 45 per cent has a half-time of one decade. As in Nordhaus (2001), we assume that doubling the steady state \(\text{CO}_2\) stock leads to 2.6 per cent output loss. This implies a value \(\pi = 0.0156 \text{[per TtCO2]}\).\(^{26}\) We assume \(\varepsilon = 0.183\) per decade, implying a global temperature adjustment speed of 2 per cent per year. This choice is within the range of scientific evidence (Solomon et al. 2007).\(^{27}\) See the Appendix for further details.

4 Equilibrium

We state first a general welfare representation for the class of policies considered in this paper. Consider policies that take the form 

\[
k_{t+1} = G_t(k_t, \Theta_t), \quad z_t = H_t(k_t, \Theta_t),
\]

where \(\Theta_t = (D_t, S_{1,t}, ..., S_{n,t})\) collects the vector of climate state variables. However, the climate affects the continuations payoffs only through the weighted sum of past emissions, as expressed in (7); for a more convenient algebra, we will replace \(\Theta_t\) by \(s_t\) below, keeping

---

\(^{25}\)Some fraction of emissions enters the ocean and biomass within a decade, so the shares \(a_i\) do not sum to unity.

\(^{26}\)Adding one TtCO2 to the atmosphere, relative to preindustrial levels, leads to steady-state damages that are about 0.79% of output. Adding up to 2.13 TtCO2 relative to the preindustrial level, leads to about 2.6% loss of output. The equilibrium damage sensitivity is then readily calculated as \((2.56 - 0.79)/(2.13 - 1) = 1.56%/\text{TtCO2}\).

\(^{27}\)In Figure 1, the main reason for the deviation from DICE 2007 is that DICE assumes an almost full \(\text{CO}_2\) storage capacity for the deep oceans, while large-scale ocean circulation models point to a reduced deep-ocean overturning running parallel with climate change (Maier-Reimer and Hasselman 1987). The positive feedback from temperature rise to atmospheric \(\text{CO}_2\) through the ocean release is essential to explain the large variability observed in ice cores in atmospheric \(\text{CO}_2\) concentrations. We note that our closed-form model can be calibrated very precisely to approximate the DICE model (Nordhaus 2007); Section 6 discusses further on the surprising prediction power of our carbon pricing formula for the DICE results. We also include a more detailed note on this issue in the supplementary material. The DICE 2013 model has updated the ocean carbon storage capacity.
in mind that the current state is equivalently described by either $\Theta_t$ or $s_t$. For given policies $G_t(k_t, s_t)$ and $H_t(k_t, s_t)$, we can write welfare in (2) as follows

$$w_t = u_t + \beta \delta W_{t+1}(k_{t+1}, s_{t+1})$$

$$W_t(k_t, s_t) = u_t + \delta W_{t+1}(k_{t+1}, s_{t+1})$$

where $W_{t+1}(k_{t+1}, s_{t+1})$ is the (auxiliary) value function.

More specifically, the payoff implications can be described through a sequence of constants $(g_\tau, h_\tau)_{\tau \geq t}$ where $0 < g_t < 1$ is the share of the gross output invested,

$$k_{t+1} = g_t y_t,$$

and $h_t$ is the climate policy variable, that is, a constant that measures the current utility-weighted marginal product of carbon, $\frac{\partial y_t}{\partial z_t} = h_t$. This, through the functional assumptions, defines the marginal product of the fossil fuel use, the carbon price, as

$$\frac{\partial y_t}{\partial z_t} = h_t(1 - g_t)y_t.$$

Similarly as $g_t$ measures the stringency of the savings policy, $h_t$ measures the stringency of the climate policy. In particular, the carbon price, $\frac{\partial y_t}{\partial z_t}$, is monotonic in policy $h_t$, which allows an interchangeable use of these two concepts.

Now, for any sequence of constants $(g_\tau, h_\tau)_{\tau \geq t}$ such that (13) and (14) are satisfied, we have a representation of welfare:

**Theorem 2** It holds for every policy sequence $(g_\tau, h_\tau)_{\tau \geq t}$ that

$$W_{t+1}(k_{t+1}, s_{t+1}) = V_{t+1}(k_{t+1}) - \Omega(s_{t+1})$$

with parametric form

$$V_{t+1}(k_{t+1}) = \xi \ln(k_{t+1}) + \tilde{A}_{t+1}$$

$$\Omega(s_{t+1}) = \sum_{\tau=1}^{t-1} \zeta_{\tau} s_{t+1-\tau},$$

where $\xi = \frac{\alpha}{1 - \alpha d}$, $\frac{\partial \Omega(s_{t+1})}{\partial z_t} = \zeta_1 = \Delta \sum_{i \in X} \frac{a_i \pi \xi}{[1 - \delta(1 - \eta_i)][1 - \delta(1 - \epsilon)]}$, $\Delta = \frac{1}{1 - \alpha d} + \Delta_u$ and $\tilde{A}_{t+1}$ is independent of $k_{t+1}$ and $s_{t+1}$.

The future cost of the emission history is thus given by $\Omega(s_{t+1})$, giving also the marginal cost of the current emissions as $\zeta_1$ that is a compressed expression for the climate-economy impacts. But, we can immediately see from Remark 1 that a closed carbon cycle leads to persistent impacts ($\eta_i = 0$ for one $i$), implying thus unbounded future marginal losses when the long-term discounting vanishes:

28We show this in Lemma 3 of the Appendix.
**Proposition 1** For a closed carbon cycle with incomplete absorption, \( \frac{\partial \Omega(s_{t+1})}{\partial z_t} \to \infty \) as \( \delta \to 1 \).

The result has strong implications for the comparison of the policies considered next.

### 4.1 Markov equilibrium policies

Theorem 2 describes continuation welfares for a class of policies, and now we proceed to a Markov equilibrium that can be found from this class. More precisely, we look for a symmetric Markov equilibrium where all generations use the same policy functions.\(^{29}\)

The Markov policies do not condition on the history of past behavior (see Maskin and Tirole, 2001), in contrast with the sustainable policies considered in Section 4.3.\(^{30}\)

Krusell et al. (2002) describe the savings policies for a one-sector model in the same parametric class with quasi-hyperbolic preferences. Our setting is more complicated since, with two-sectors, the policies for the sectors can be either strategic substitutes or complements; however, the Brock-Mirman (1972) structure for the consumption choice and exponential productivity shocks from climate change eliminates such interactions, and thus the savings and climate policies become separable.\(^{31}\)

Each generation takes the future policies, captured by constants \((g, h)\) in (13)-(14), as given and chooses its current savings to satisfy

\[
u'_t = \beta \delta V'_{t+1}(k_{t+1}),\]

where \(u'_t\) denotes marginal consumption utility and function \(V(\cdot)\) from Theorem 2 captures the continuation value implied by the equilibrium policy.

**Lemma 1** *(savings)* The equilibrium investment share \(g = k_{t+1}/y_t\) is

\[
g^* = \frac{\alpha \beta \delta}{1 + \alpha \delta (\beta - 1)}. \tag{15}\]

\(^{29}\) Even though there can be technological change and population growth, the form of the objective, combined with (2), ensures that there will be an equilibrium where the same policy rule will be used for all \(t\).

\(^{30}\) We will construct a natural Markov equilibrium where policies have the same functional form as when \(\beta = 1\). Moreover, Iverson (2014) shows for this model that the Markov equilibrium considered here is the unique limit of a finite horizon equilibrium. For multiplicity of equilibria in related settings, see Krusell and Smith (2003) and Karp (2007).

\(^{31}\) In the online Appendix, we develop a three-period model with general functional forms to explicate the interactions eliminated by the parametric assumptions.
The proof of the Lemma is a straightforward verification exercise following from the first-order condition. If future savings could be dictated today, then \( g^{\beta=1} = \alpha \delta \) for future decision-makers would maximize the wealth as captured by \( W_{t+1}(k_{t+1}, s_{t+1}) \); however, equilibrium \( g^* \) with \( \beta < 1 \) falls short of \( g^{\beta=1} = \alpha \delta \) because each generation has an incentive to deviate from this long-term plan due to higher impatience in the short run (Krusell et al., 2002).

Consider then the equilibrium choice for the fossil-fuel use, \( z_t \), satisfying

\[
0 = \frac{\partial y_t}{\partial z_t} = \beta \delta \frac{\partial \Omega(s_{t+1})}{\partial z_t}.
\]

The optimal policy thus equates the marginal current utility gain from fuel use with the change in equilibrium costs on future agents. Denote the equilibrium carbon price by \( \tau_t^{\beta \delta} (= \partial y_t/\partial z_t) \). Given Theorem 2, carbon price \( \tau_t^{\beta \delta} \) can be obtained:

**Proposition 2** The equilibrium carbon price is

\[
\tau_t^{\beta \delta} = h^*(1 - g^*)y_t \tag{16}
\]

\[
h^* = \Delta \sum_{i \in I} \frac{\beta \delta a_i \pi \varepsilon}{1 - \delta (1 - \eta_i)(1 - \delta (1 - \varepsilon))} \tag{17}
\]

\[
\Delta = \left( \frac{1}{1 - \alpha \delta} + \Delta_u \right)
\]

When \( y_t \) is known, say \( y_t = 2010 \), the carbon policy for \( t = 2010 \) can be obtained from (16), by reducing fossil-fuel use to the point where the marginal product of \( z \) equals the externality cost of carbon. Again, if future policies could be dictated today, externality cost would be higher \( h^{\beta=1} > h^* \), dominating the effect from higher savings; that is, \( h^*(1 - g^*) \) is increasing in \( \beta \).\(^{32}\)

To obtain the current externality cost of carbon intuitively, that is, the social cost of carbon emissions \( z_t \) as seen by the current generation, consider the effect of damages \( D_{t+\tau} \) on utility in period \( t+\tau \). Recall that the consumption utility is \( \ln(c_{t+\tau}) = \ln((1-g)y_{t+\tau}) = \ln(1-g) + \ln(y_{t+\tau}) \) so that, through the exponential output loss in (6), \( \partial \ln(c_{t+\tau})/\partial D_{t+\tau} = -1 \). As there is also the direct utility loss, captured by \( \Delta_u \) in (6), the full loss in utils at \( t + \tau \) is

\[
-\frac{d u_{t+\tau}}{d D_{t+\tau}} = 1 + \Delta_u.
\]

\(^{32}\)In Section 4.3 we return to this issue by considering commitment policies that be sustained in equilibrium.
But, the output loss at \( t + \tau \) propagates through savings to periods \( t + \tau + n \) with \( n > 0 \),

\[
- \frac{du_{t+\tau+n}}{DD_{t+\tau}} = \alpha^n,
\]

leading to the full stream of losses in utils, discounted to \( t + \tau \),

\[
- \sum_{n=0}^{\infty} \delta^n \frac{du_{t+\tau+n}}{DD_{t+\tau}} = \frac{1}{1 - \alpha \delta} + \Delta_u = \Delta.
\]

The full loss of utils per increase in temperatures as measured by \( D_{t+\tau} \) is thus a constant given by \( \Delta \) for any future \( \tau \), giving the social cost of carbon emissions \( z_t \) at time \( t \), appropriately discounted to \( t \), as

\[
\sum_{\tau=1}^{\infty} \delta^\tau \frac{du_{t+\tau}}{dz_t} = \Delta \sum_{\tau=1}^{\infty} \beta \delta^\tau \frac{dD_{t+\tau}}{dz_t} = \Delta \sum_{\tau=1}^{\infty} \beta \delta^\tau \sum_{i \in I} \frac{\beta a_i \pi \varepsilon}{\varepsilon - \eta_i} \sum_{\tau=1}^{\infty} \delta^\tau (1 - \eta_i)^\tau - \delta^\tau (1 - \varepsilon_j)^\tau = \Delta \sum_{\tau=1}^{\infty} \beta \delta^\tau \pi a_i \varepsilon \frac{1 - \delta (1 - \eta_i)}{1 - \delta (1 - \varepsilon_j)}.
\]

This is exactly the value of \( h^* \). Thus, in equilibrium, the present-value utility costs of current emissions remain constant at level \( h^* \). However, since this cost is weighted by income in (16), the equilibrium carbon price increases over time in a growing economy.

The Markov equilibrium carbon price depends on the delay structure in the carbon cycle captured by parameters \( \eta_i \) and \( \varepsilon \). Carbon prices increase with the damage sensitivity \( (\partial h/\partial \pi > 0) \), slower carbon depreciation \( (\partial h/\partial \eta_i < 0) \), and faster temperature adjustment \( (\partial h/\partial \varepsilon > 0) \). Higher short- and long-term discount rates both decrease the carbon price \( (\partial h/\partial \beta > 0; \partial h/\partial \delta > 0) \). Consistent with Proposition 1, the carbon price rises sharply if the discount factor comes close to one, \( \delta \to 1 \), and if some box has slow depreciation, \( \eta_i \to 0 \)\(^{33}\).

### 4.2 Markov vs. revealed-preference policies

The Markov policy for pricing emissions deviates from the principle that all investments in the economy, public or private, should earn the same return. Formally, this benchmark

\(^{33}\)If carbon depreciates quickly, \( \eta_i \gg 0 \), then the carbon price will be less sensitive to the discount factor \( \delta \). Fujii and Karp (2008) conclude that the mitigation level is not very sensitive to the discount rate. Their representation of climate change can be interpreted as one in which the effect of \( CO_2 \) on the economy depreciates more than 25 per cent per decade. This rate is well above the estimates for \( CO_2 \) depreciation in the natural-science literature; however, induced adaptation may lead to similar reduction in damages.
can be obtained by considering a planner who “respects” the Markov equilibrium savings decisions and uses the capital returns for evaluating the climate policy. More precisely, the equilibrium defines a utility-discount factor $0 < \gamma < 1$ for consumption, obtained from

$$u'_t = \gamma u'_{t+1} R_{t,t+1}$$

where $R_{t,t+1}$ is the capital return between $t$ and $t + 1$. Thus,

$$\gamma = \frac{u'_t}{u'_{t+1} R_{t,t+1}} = \frac{c_{t+1}}{c_t} \frac{k_{t+1}}{\alpha y_{t+1}} = \frac{g}{\alpha}. \quad (18)$$

In the Markov equilibrium where $g = g^*$, we have

$$\gamma^* = \frac{\beta \delta}{1 + \alpha \delta (\beta - 1)}. \quad (19)$$

This is the geometric utility discount factor that is consistent with the efficiency of the equilibrium consumption stream: a fictitious Ramsey planner who has consistent preferences and discounts with $\gamma^*$ would find the equilibrium policy $g^*$ optimal. This “observational equivalence” has been noted in a one-sector growth setting by Barro (1999); see also Krusell et al. (2002). However, with two sectors, for the planner to represent the economy, the same utility discount factor $\gamma = \gamma^*$ must be used for both savings and climate policies. Thus, we find the social cost of carbon for a planner who discounts with $\gamma^*$. Since this defines the full externality cost of emissions for such a planner, we arrive at the definition of the Pigouvian tax for the consistent preferences discounting $\gamma^*$.

**Proposition 3** (revealed-preference Pigouvian tax) The optimal carbon price, $\tau^\gamma_t$, for a Ramsey planner who discounts utilities with geometric discount factor $\gamma = \gamma^*$ equals the consumption-weighted net present value of future marginal utility losses from emissions $z_t$. Value $\tau^\gamma_t$ is given by

$$\tau^\gamma_t = h^\gamma (1 - g) y_k$$

$$h^\gamma = \Delta^\gamma \sum_{i \in I} \frac{\gamma \pi a_i \varepsilon}{[1 - \gamma(1 - \eta_i)][1 - \gamma(1 - \varepsilon)]} \quad (20)$$

$$\Delta^\gamma = \frac{1}{1 - \alpha \gamma} + \Delta_u. \quad (21)$$

The Markov equilibrium tax, in Proposition 2, is different from the one in Proposition 3, a result that we address in detail shortly. Note first that the planner identified here maximizes a weighted sum of the utility sequence, and in this sense the economy that implements the revealed-preference Pigouvian tax is on the efficiency frontier. It is useful
to state this benchmark explicitly since it formalizes the rationale of Pigouvian externality pricing based on the capital returns in the economy. The (fictitious) planner chooses emissions so that today’s marginal carbon product, \( MCP_t \), equals the sum of future marginal carbon damages caused by current emissions, \( MCD_{t,T} \), for \( T > t \), discounted to the present by the marginal rate of substitution, \( MCP_t = \sum_{T>t} MCD_{t,T} / MRS_{t,T} \).

Since the planner implements a time-consistent path, capital savings lead to allocations satisfying \( MRS_{t,T} = R_{t,T} \), and the fictitious planner’s optimal carbon price equals the net present value of damages \( MCP_t = \tau^\gamma_t \). The Markov equilibrium deviates from this benchmark since the equilibrium compound capital return between \( t \) and some future period \( T > t + 1 \), no longer reflects how the current policy-maker sees the consumption trade-offs: \( MRS_{t,T} < R_{t,T} \), the capital returns are excessive. As a result, the Markov climate policy is tighter than the one using capital returns: \( MCP_t = \sum_{T>t} MCD_{t,T} / MRS_{t,T} > \sum_{T>t} MCD_{t,T} / R_{t,T} \). We establish now the precise conditions for the the policies to differ:

**Proposition 4** For \( \beta, \delta < 1 \), the Markov equilibrium carbon price strictly exceeds the revealed-preference Pigouvian level if climate change delays are sufficiently long. Formally, ratio \( \tau^\beta_t / \tau^\gamma_t \) is continuous in parameters \( \beta, \delta, \eta_i, \varepsilon, a_i, \) and \( \gamma \). Evaluating at \( \gamma = \gamma^* \), \( \beta < 1 \), \( \eta_i = \varepsilon = 0 \),

\[
\tau^\beta_t > \tau^\gamma_t.
\]

If the climate system is sufficiently persistent, the Markov decision-makers value the commitment to future utility impacts. From another perspective, it is well known that when \( \beta < 1 \) the future equilibrium savings are lower than preferred from the current generation’s point of view (Krusell et al. 2002). There is thus a capital market distortion, implying higher future capital returns than what the current generation would like to see. The revealed-preference Pigouvian tax uses those distorted returns to obtain the present value of climate impacts, and thus identifies a wrong cost-benefit ratio for the current emissions; this links with the well-know result in cost-benefit analysis that the distorted capital returns do not identify the correct social returns for public investments (Lind, 1982; Dasgupta, 2008). The true return on climate policies is higher if the climate asset is sufficiently persistent; the equilibrium carbon price formula incorporates the social value of this persistence. The distortion identified here has no bound in the following sense:

**Proposition 5** For a closed carbon cycle with incomplete absorption and \( \beta < 1 \): \( \tau^\beta_t / \tau^\gamma_t \rightarrow \infty \) as \( \delta \rightarrow 1 \).
When no carbon leaves the system, a fraction of the climate impacts is persistent, which drives the Markov price to infinity while the Pigouvian price remains bounded.

But is it good or bad for the overall welfare that the Markov policies lead to “over-investment” in the climate asset? Would it be better to require that the same return requirement should be used both for capital and climate investments, for example, through an institutional constraint? We show next that such a requirement, which is typically invoked in cost-benefit analysis, does not improve welfare here. We must first address the meaning of welfare in this context. Since we treat agents in different periods as distinct generations (as in Phelps and Pollak, 1968), the multi-generation Pareto optimality is a legitimate welfare concept (as, e.g., in Caplin and Leahy, 2004) for considering whether policy measures can improve welfare above that in the Markov equilibrium. We provide such a comprehensive welfare analysis for an equivalent model in Gerlagh & Liski (2011); here we bring the essence of the welfare impacts.

From the perspective of the current generation, future savings and emission levels are optimal if they are consistent with the long-term time preference \( \delta \), that is, if \( g = \alpha \delta \) and \( h = h^\gamma = \delta \) where \( h^\gamma \) is defined in Proposition 3; then future agents would behave as if they were consistent with the present-time long-term preferences. This thought-experiment gives a clear benchmark against which we can test how policy proposals affect current welfare through future policies.

**Lemma 2** For \( \beta \neq 1 \) and any given \( \tau > t \),
\[
\frac{\partial w_t}{\partial g_\tau} > 0 \text{ iff } g_\tau < \alpha \delta \\
\frac{\partial w_t}{\partial h_\tau} > 0 \text{ iff } h_\tau < h^\delta.
\]

Since the equilibrium policies depart from those optimal for the long-run preference \( \delta \), any policy that manages to take the decision variables closer to the long-run optimal levels increases current welfare. It turns out that imposing the stand-alone Pigouvian carbon tax principle implies a correction in the wrong direction.

**Proposition 6** For slow climate change, implementing \( \partial y_t/\partial z_t = \tau^*_t \) from period \( t \) onwards implies a welfare loss for generation \( t \) vis-à-vis the Markov equilibrium.

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34See Bernheim and Rangel (2009) for an alternative concept, and its relationship to the Pareto criterion. The Pareto criterion may not be reasonable when the focus is on the behavioral anomalies at the individual level.

35For completeness, these results are reproduced for the current climate-economy model in our longer working paper version Gerlagh & Liski (2012).
The remarkable feature of the above proposition is that the carbon pricing policy based on “revealed-preference” returns decreases welfare, not as a second-order effect, but as a first-order effect. This is unfortunate since it would simplify the cost-benefit analysis if the economy’s revealed preference for discounting the distant future could be used. However, policies that coordinate savings decisions over time have an impact on the time preference “revealed” by the economy. We consider such policies in the next Section and, then, in the quantitative analysis we show that the equilibrium coordination of savings can restore the revealed-preference approach for climate policies.

4.3 Sustainable policies

In Theorem 2, the welfare representation holds not only for the Markov policies but for any arbitrary policy sequence \((g_\tau, h_\tau)_{\tau>t}\) that future decision-makers might adopt. Clearly, the welfare can be improved if we can bring the future policies closer to the choices that the current decision-maker would like to see in the future. For such a policy plan to be self-enforcing, the future policy-makers would need to voluntarily follow suit by anticipating similar collaboration after their time, conditional on “good” policy-making in the past. Such dependencies between policies over periods can be constructed in a myriad of ways. We look for a self-enforcing policy proposal that is the best achievable policy among symmetric policies.

Consider a policy pair \((\hat{g}, \hat{h})\) that the current generation would like to propose for all generations, including itself. Symmetry requires that \((\hat{g}, \hat{h})\) is the same for all affected generations. From Lemma 2, generation \(t\) would like to propose to all future generations the decision rule \(g^\delta\) and \(h^\delta\) but achieving this requires that these policies are followed also at \(t\), which is ruled out by the current incentive constraints: \((g^\delta, h^\delta)\) does not maximize \(w_t\); the policy maximizes \(W_{t+1}\). But, also from Lemma 2, the current generation is willing to give up part of its consumption, by increasing \(g\) and \(h\), beyond their Markov equilibrium levels, anticipating that all subsequent decision-makers will follow suit when facing the same decision.\(^{37}\)

\(^{36}\)In a different context, Bernheim and Ray (1987) also show that, in the presence of altruism, consumption efficiency does not imply Pareto optimality.

\(^{37}\)Roemer (2010) defines a Kantian equilibrium where each subject presumes that all other subjects follow the same rule; for a static economy, Roemer shows that the Kantian conjecture leads to an efficient outcome. In our context, a Kantian policy rule would demand that each generation sets a policy pair \((g, h)\) that it would like to see for both future and past generations. But in our setting, time flows only forward so we derive only forward-looking policy rules. Moreover, our policy is self-enforcing.
Policy \((\hat{g}, \hat{h})\) that we define is sustainable; it is self-enforcing, and identifies a well-defined optimal symmetric “contract” that can be maintained as proposal for the future at each \(t\). We define policies \(\hat{g}\) and \(\hat{h}\) independently. Formally, at time \(t\), the proposal for savings, for any sequence of \(h_{\tau}\) \((\tau \geq t)\), is

\[
\hat{g}_t \in \{ \text{arg max}_{g \in [0,1]} w_t | g_{\tau} = g \text{ for all } \tau \geq t \}.
\]

Building on Theorem 2, we show in Appendix that the set is well defined. The equilibrium proposal comes from

\[
\hat{g} \in \cap_{t=1}^{\infty} \{ \hat{g}_t \}, \tag{22}
\]

or if no joint proposal emerges, \(\hat{g}\) is taken to be the Markov rule \(g^*\). Similarly, without conditioning on \(g\), the proposal for the climate policy is

\[
\hat{h}_t \in \{ \text{arg max}_{h \in \mathbb{R}} w_t | h_{\tau} = h \text{ for all } \tau \geq t \} \Rightarrow \hat{h} \in \cap_{t=1}^{\infty} \{ \hat{h}_t \}. \tag{23}
\]

If the set is empty, we take \(\hat{h} = h^*\).

**Proposition 7**  
Sustainable policy \((\hat{g}, \hat{h})\) satisfies

\[
g^\delta > \hat{g} > g^* \\
h^\delta > \hat{h} > h^*
\]

where \(\hat{g}\) is unique. Moreover, \(\hat{h} = h^*\) if \(A_t \neq A_{\tau}\) for \(\tau > t\). The sustainable imputed discount factor is

\[
\hat{\gamma} = \frac{\beta \delta}{1 + \alpha \delta (\beta - 1) + (1 - \alpha \delta)(\beta - 1) \delta}
\]

such that \(\gamma^* < \hat{\gamma} < \delta\).

The unique sustainable saving rule is \(\hat{g} = \alpha \hat{\gamma};\) all agents agree on these higher-than-Markov savings. In contrast, if technology \(A_t\) changes over time, it is generally not possible to find any other “contractible” symmetric policy rule \(\hat{h}\) than the Markov rule \(h^*\). But there is still an impact on the carbon price: sustainable savings mitigate the capital market distortion, thereby reducing the distortion correction in the carbon price. If savings can be co-ordinated in the sense discussed here, the Pigouvian tax based on revealed discounting (i.e., the imputed tax using \(\hat{\gamma}\)) and the equilibrium carbon price are close to equal, as our quantitative assessment shows. They are also equal in the following limit:
Proposition 8 For low long-term discounting, $\delta \to 1$, zero time-discounting is sustainable: $\hat{\gamma} \to 1$.

For a closed carbon cycle (with incomplete absorption), the sustainable imputed carbon price becomes then unbounded, $\tau^{\hat{\gamma}}_t \to \infty$, as is the case for the Markov price.

5 Quantitative assessment

To evaluate the quantitative significance of the conceptual results, we exploit the closed-form price formulas — given the structure of policies, the initial carbon price level is a function of the income level and the carbon cycle parameters. Reasonable choices for the climate-economy parameters and consistent preferences ($\beta = 1$) can reproduce the carbon price levels of the more comprehensive climate-economy models such as DICE (Nordhaus, 2007).

We then introduce a difference between short- and long-term discounting, $\beta < 1$, while keeping the macroeconomy, i.e., capital savings, observationally equivalent to the Nordhaus case. The quantitative evaluation is thus structured such that we control for the capital savings, using the relationship between equilibrium savings $g$ and discount factors $\beta, \delta$— this allows keeping the Nordhaus case as a well-justified benchmark and experimenting how “the same economy” prices carbon differently for an increasing difference between short- and long-run discount rates.

The model is decadal (10-year periods) and year ‘2010’ corresponds to period 2006-2015. We set $\Delta_u = 0$. We take the Gross Global Product as 600 Trillion Euro [Teuro] for the decade, 2006-2015 (World Bank, using PPP). The capital elasticity $\alpha$ follows from the assumed time-preference structure $\beta$ and $\delta$, and observed historic gross savings $g$. As a base-case, we consider net savings of 25% ($g = .25$), and a 2.7 per cent annual pure rate of time preference ($\beta = 1, \delta = 0.761$), consistent with $\alpha = g/\delta = 0.329$.

5.1 Assessment of Markov policies

The parameter choices together with our carbon cycle result in a consistent-preferences Pigouvian carbon price of 7.1 Euro/tCO$_2$, equivalent to 34 USD/tC, for 2010. This

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38 We can also reproduce the carbon tax time path of DICE when the energy sector of our model is specified and calibrated in detail; see our working paper Gerlagh&Liski (2012).

39 The period length could be longer, e.g., 20-30 years to better reflect the idea that the long-term discounting starts after one period for each generation. We have these results available on request.

40 Note that 1 tCO2 = 3.67 tC, and 1 Euro is about 1.3 USD.
number is very close to the level found by Nordhaus. Consider then the determinants of this number in detail.

We can decompose the carbon price into three contributing parts. First, consider the one-time costs assuming full immediate damages ($ID$) taking place in the immediate next period,

$$ID = \beta \delta \Delta \pi (1 - g) y_t.$$  

This value is multiplied by a factor to correct for the persistence of climate change due to slow depreciation of carbon in the atmosphere, the persistence factor ($PF$),

$$PF = \sum_{i \in I} \frac{a_i}{1 - \delta (1 - \eta_i)};$$

which we then multiply by a factor to correct for the delay in the temperature adjustment, the delay factor ($DF$),

$$DF = \frac{\epsilon}{1 - \delta (1 - \epsilon)}.$$ 

Table 2 below presents the decomposition of the carbon tax for a set of short- and long-term discount rates such that the economy’s savings policy remains the same. The first row reproduces the efficient carbon price case assuming consistent preferences when the annual utility discount rate is set at 2.7 per cent: this row presents the carbon price under the same assumptions as in Nordhaus (2007). Keeping the equilibrium time-prefeference rate at 2.7 per cent per year, thus maintaining the savings rate at a constant level (reported also in Table 1 of the Introduction), we move to the Markov equilibrium by departing the short- and long-term discount rates, presented in the first and second columns.

We invoke Weitzman’s (2001) survey for obtaining some guidance in choosing the short- and long-run rates. In Weitzman, discount rates decline from 4 per cent for the immediate future (1-5 years) to 3 per cent for the near future (6-25 years), to 2 per cent for medium future (26-75 years), to 1 per cent for distant future (76-300), and then close to zero for far-distant future. Roughly consistent with Weitzman and our 10-year length of one period, we use the short-term discount rate close to 3 per cent, and the long-term rate at or below 1 per cent. This still leaves degrees of freedom in choosing the two rates $\beta \delta$ and $\delta$; we choose them to match the savings rate of 25 per cent and thus the macroeconomic performance in Nordhaus (2007). That is, we choose $\beta$ and $\delta$ to maintain the equilibrium utility discount factor at $\gamma = 0.76$ (2.7 per cent annual discount rate). Since the equilibrium utility discount rate remains at 2.7 per cent, the economy remains observationally equivalent to that in Nordhaus ($g = .25$)\textsuperscript{41}

\textsuperscript{41}For example, 3 per cent short run and 1 per cent long run annual rates correspond to $\beta = .788$ and
Table 2: Decomposition of the carbon price [Euro/tCO2] year 2010. \( ID = \) immediate damages, \( PF = \) persistence factor, \( DF = \) delay factor, Carbon price = \( ID \times PF \times DF \). Parameter values in text.

We obtain a radical increase in the carbon price as the long-term discounting decreases, while savings remain unchanged from one set of preferences to the next. Note that, by construction, the Nordhaus number 7.1 EUR/tCO2 is the revealed-preference and thus the non-optimal tax for the hyperbolic discounting cases; since all equilibria have the same equilibrium discount rate, the imputed (‘revealed preference’) tax remains constant. The highest equilibrium carbon tax, 133 EUR/tCO2, corresponds to the case where the long-run discounting is as proposed by Stern (2006); this case also best matches Weitzman’s values. For reference, we report the Stern case where the long-term discounting at .1 per cent holds throughout; the carbon price takes a value of 174 EUR/tCO2, and gross savings cover about 33 per cent of income. Thus, the Markov equilibrium closes considerably the gap between Stern’s and Nordhaus’ carbon prices, without having unrealistic by-products for the macroeconomy\(^{42}\).

The decomposition of the carbon price is revealing. Leaving out the time lag between \( CO_2 \) concentrations and the temperature rise amounts to replacing the column \( DF \) by 1. When preferences are consistent (the first line), abstracting from the delay in temperature adjustments, as in Golosov et al. (2014), doubles the carbon price level. For hyperbolic discounting, as expected, the persistence of impacts, capturing the commitment value of climate policies, contributes significantly to the deviation between the imputed and

\( \delta = .904 \). See the supplementary material for all numerical values.

\(^{42}\)The deviation between the Markov (thus Nordhaus) and Stern savings can be made extreme by sufficiently increasing the capital share of the output that gives the upper bound for the fraction of \( y_t \) saved; close to all income is saved under Stern preferences as this share approaches unity (Weitzman, 2007). However, with reasonable parameters such extreme savings do not occur, as in Table 2.
Markov equilibrium prices.

Table 2 quantifies the economic substance of appropriately accounting for the wedge between the marginal rate of substitution and transformation: when the long-run discount rate declines, the future equilibrium saving rate falls below the one the current generation would like to see. The greater is this wedge, the larger is the gap between the currently optimal carbon price based on the MRS, and the revealed-preference Pigouvian tax based on the MRT.

5.2 Assessment of sustainable policies

The “savings contract” from Proposition 7 deals with the main distortion arising from time-changing discounting in our intergenerational policy game. Coordination of savings has strong implications for equilibrium discounting as Table 3 shows: the sustainable imputed discount rate falls quickly below the Markov level that stays at 2.7 per cent (as in Table 2). The revealed-preference Pigouvian price, reported in the last column of Table 3 is based on this new lower equilibrium rate and thus increases strongly as a result of savings co-ordination. The assessment confirms that, if savings are coordinated, using aggregate savings information for obtaining the appropriate externality price gives close to the correct value, that is, the best-responding equilibrium carbon tax\(^{43}\)

<table>
<thead>
<tr>
<th>annual discount rate</th>
<th>sustainable savings rate</th>
<th>carbon price</th>
</tr>
</thead>
<tbody>
<tr>
<td>short-term</td>
<td>long-term</td>
<td>equilibrium</td>
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<tr>
<td>.027</td>
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<td>.033</td>
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<td>.035</td>
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<td>.0062</td>
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<tr>
<td>.037</td>
<td>.001</td>
<td>.0013</td>
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<td>.001</td>
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</tbody>
</table>

Table 3: Sustainable carbon prices based on \(\hat{g} = \alpha \hat{\gamma}\) and \(\hat{h} = h^*\) and the imputed Pigouvian externality price [Euro/tCO2] year 2010. Parameter values in text.

\(^{43}\)We assume for the sustainable equilibrium carbon prices that \(\hat{h} = h^*\), which from Proposition 7 is the equilibrium when technology \(A_t\) changes over time.
6 Discussion

To obtain transparent analytical and quantitative results in a field that has been dominated by simulation models, we exploit strong functional assumptions. First, building on Brock-Mirman (1972) we assumed that income and substitution effects in consumption choices over time cancel out, leading to policies for savings and carbon prices that are separable. With general functional forms, climate policies can generate income effects influencing future savings, thereby creating deeper linkages between the two policies. Based on a three-period extension to general functional forms, presented in the online Appendix, we discuss below the effects that are ruled out by the assumptions in the main analysis. Second, we assumed a linearized model for carbon diffusion that might not well describe the relevant dynamics when the system is far off the central path — that is, non-linearities captured by more complicated climate simulation models may be important. Finally, the quasi-hyperbolic discount functions are only rough approximations of more general discount functions. Given the parametric class for preferences and technologies, it is possible to solve this model for an arbitrary sequence of discount factors; this extension is provided in Iversion (2014). The flexible discounting does not change the conceptual substance matter in a material way, although the quantitative evaluations can depend on the added flexibility. Yet, currently, there is no evident data for the path of the time-preferences that would call for the flexible formulation. We now briefly discuss how results may be expected to change for other functional forms of utility, production and climate change.

6.1 Sensitivity of policies under geometric discounting

Before assessing the changes in strategic interactions when the functional forms are more general, we must assess the sensitivity of climate policies to functional forms in a context where current and future planners do not strategically interact but where policies follow a time-consistent planning. To address the sensitivity of carbon prices with geometric discounting, we devised a Monte Carlo experiment for testing how well the closed-form carbon price formula that we have developed, building on the specific functional assumptions, predicts the carbon price of a benchmark simulation model, DICE 2007. This benchmark model assumes a more general parametric class for preferences and tech-

\[44\] There is an emerging literature providing revealed-preference evidence for very long discount rates (see Giglio et al. 2014). However, this is not direct evidence on the rate of time preference and cannot be applied for calibration purposes as such.
nologies, and also features non-linearities of the climate system. Assuming geometric
discounting and drawing parameters from pre-determined distributions for all key pa-
rameters in DICE, including those that appear in our formula as well as those not in
our formula, we found that the formula explains 99 per cent of the DICE variation in
the carbon price. This suggests that the loss of generality from not including (i) the
deeper linkages between policies, that capture mainly income and substitution effects in
consumption over time, and (ii) the non-linearities of climate change has limited conse-
quences for the carbon price results.

Obviously, the experiment has its limitations. The DICE2007 model itself is very
much reduced form, and other models may present different outcomes. Specifically, DICE
does not feature other greenhouse gases; it does not consider endogenous technological
change; and it has only one representative consumer. But, whereas these features stress
the limitations of DICE, they also make it the best candidate as a benchmark for our
results; it is both the most widely used integrated assessment model, and it has the same
set of outcome variables as our model has. The second qualification is that, for very
low discounting, close to zero, in our model carbon prices increase without bound. The
result is useful since it identifies the impact-response elements that contributes to the
rapidly increasing social cost; that is, the share of persistent carbon and its decay rate.
This same mechanism can also be identified using DICE. But the social cost can remain
bounded for other reasons: if we assume perpetual economic growth without bound, and
if the elasticity of marginal utility exceeds one, then even with zero discounting the value
of the full future consumption stream and the value of damages expressed as a fraction of
output are both bounded. On the other hand, if the economy converges so that income is
bounded from above, or if the elasticity of marginal utility is less than one, or if climate
change damages enter utility directly and separable from consumption, then Proposition
1 can be shown to hold also for a non-logarithmic utility. Thirdly, when the relevant time
horizons extend to centuries, it may not be reasonable to think that the same fraction
of the output is lost from a given temperature increase independently of when the loss
takes place. After centuries, the economy may have adapted to climate change; moreover,
climate-change induced biodiversity losses may have to be conceptualized differently.

Yet, the evidence from the above experiment is suggestive that tractable climate-
economy models can be very good approximations for the more comprehensive models.

45See Figure 2 in the note titled "SCC Formula" of the supplementary material. The note summarizes
the results and documents the experiment. For conciseness, the note presents a continuous-time version
of the carbon price formula; the note is self-contained.
Since the carbon price is a closed-form function of the climate-economy fundamentals, such models can prove useful in addressing the consequences of uncertainties related to the fundamentals in a direct way. Our reduced-form carbon cycle and damage representations assumed no uncertainty, although great uncertainties describe both the climate system parameters as well as the impacts of climate change. Golosov et al. (2014) make progress in this direction showing that the optimal polices are robust to impact uncertainty; this effectively leads to rewriting of the carbon price formula in expected terms. Iverson (2014) shows the robustness of the Markov equilibrium policy rules in a stochastic Markov equilibrium with multiple stochastic parameters. Furthermore, it is possible to construct carbon price distributions directly using the tractable carbon price formula and the primitive uncertainties regarding its determinants to decompose their respective contributions to carbon price uncertainty.\footnote{The analysis of uncertainties is beyond the scope of the current paper; however, see Figure 3 in the supplementary note "SCC Formula" for an illustration of the approach.}

### 6.2 Sensitivity of strategic carbon policies

Moving to a general description of preferences, technologies and climate change, opens new opportunities to strategically influence the future policies by current decisions. In a stylized but general three-period model (see the online Appendix), we can show that a higher elasticity of marginal utility leads to laxer climate today, as current planners foresee that future planners will tend to compensate a current increase in emissions through changes in savings. Today’s climate policies and future savings become strategic substitutes, which tends to lower the equilibrium carbon price today. In addition, the strategic substitutability of policies depends on the interaction between damages and output. A less-than-proportional increase of damages implies that current emissions have less of an effect on future returns on capital investments, and, thus, current emissions become less of a substitute for future savings. Therefore, we find laxer climate policies when damages are less dependent on output levels.

Yet, these effects are indirect and we have no reason to believe that they are quantitatively substantial.\footnote{Iverson, in a revision of his (2012) manuscript, follows up on our analysis and develops a numerical model to conclude: "Nevertheless, in all cases the quantitative effect [of a different parametric form] is tiny - on the order of one one-thousandth the magnitude of the initial period perturbation."} We believe there is more scope for strategically guiding future decision makers by investing in specific capital stocks, including technology. Generally, we expect that if capital and emissions are complementary in production, then planners...
with quasi-hyperbolic preferences will tend to invest less in capital, as it commits future planners to increased emissions. But, specific types of capital that substitute for emissions, such as investments in clean energy or clean energy R&D, will attract larger investments from a planner who wishes to commit future planners to lower emissions. In spirit of Gul and Pesendorfer (2001), decision-makers may want to expend resources to remove alternatives (such as cheap fossil fuels) from the future choice sets.

We can also assess how the details of climate change damages are expected to modify results. If marginal damages tend to increase with past emissions, emissions will be strategic substitutes over time, and the current generations can strategically increase their own emissions expecting future generations to reduce theirs in response. The extreme case of such a scenario is one in which there is a known catastrophe threshold. Suppose that climate change is moderate up to levels of cumulative emissions in the range of four thousand Gigaton of $CO_2$, after which a trigger sets in dangerous climate change. Given such known threshold, each generation can freely add their emissions, as long as the threshold is not reached, as on the margin future policies will offset current emissions one-to-one. Similarly, we may consider different greenhouse gases having different lifetimes, and thereby, different commitment value. Long-lived gases, such as $N_2O$, will typically provide larger commitment, as compared to short-lived gases such as methane.

7 Concluding remarks

September 2011, the U.S. Environmental Protection Agency (EPA) sponsored a workshop to seek advice on how the benefits and costs of regulations should be discounted for projects with long horizons; that is, for projects that affect future generations. The EPA invited 12 academic economists to address the following overall question: “What principles should be used to determine the rates at which to discount the costs and benefits of regulatory programs when costs and benefits extend over very long horizons?” In the background document, the EPA prepared the panelists for the question as follows: “Social discounting in the context of policies with very long time horizons involving multiple generations, such as those addressing climate change, is complicated by at least three factors: (1) the “investment horizon” is significantly longer than what is reflected in observed interest rates that are used to guide private discounting decisions; (2) future generations without a voice in the current policy process are affected; and (3) compared to shorter time horizons, intergenerational investments involve greater uncertainty. Understanding these issues and developing methodologies to address them is of great importance given
the potentially large impact they have on estimates of the total benefits of policies that impact multiple generations.”

In this paper, we have developed a methodology for addressing the over-arching question posed above and a quantitative evaluation. We developed a general-equilibrium carbon price policy grounded in social preferences, adding a new approach to the literature that so far has focused on carbon prices based on the evaluation of future damages by use of capital returns or ethical guidelines. Our analysis incorporates the principle, often invoked in practical program evaluations, that the time-discounting rate should depend on the time horizon of the project. In general equilibrium, which is the approach needed for climate policy evaluations, time-changing discount rates drive a wedge between the marginal rate of substitution and transformation, stipulating a correction to the carbon price resulting from the evaluation of future damages based on capital returns. That is, we showed that capital returns do not reveal the climate-relevant time-pattern of social preferences, even though the capital investments are based on the same social preferences.

The resulting tool for policy purposes is a carbon pricing formula that compresses the relevant elements of the climate and the economy — while it is not a substitute for the comprehensive climate-economy models, the formula identifies the contributions of the key elements to optimal carbon prices and allows discussing them transparently. For discount factors consistent with those in the literature were used to show that the equilibrium correction to the standard Pigouvian pricing principle is significant. However, there is very limited solid empirical evidence on the time-structure of social preferences over long time horizons. Our study shows the relevance of such information, as it has a large effect on the evaluation of currently observed energy-use patterns. The carbon price directly impacts the estimate of “genuine savings” that are calculated by, for example, the World Bank. Currently valued at 20$/tC, the World Bank estimates the “negative savings” due to CO₂ emissions at 0.3 per cent of GDP for the US, and 1.1 per cent for China. Using a 100$/tC carbon price (21EUR/tCO₂) derived from a moderate quasi-hyperbolic preference structure, will increase the estimate for the negative savings to above 1 percent for the US and above 5 per cent for China. More in general, the formula allows policy-makers to experiment with their prescriptive views on longer-term discounting to see the effect on the optimal carbon price.
References


Appendix

Proof of Theorem 1

Given the sequence of climate variables — carbon stocks $S_{i,t}$ and damages $D_t$ — that we developed in the text, it is a straightforward matter of verification that future damages
depend on past emissions as follows:

\[
S_{t,t} = (1 - \eta_i)^{t-1}S_{t,1} + \sum_{\tau=1}^{t-1} a_i(1 - \eta_i)^{\tau-1}z_{t-\tau} \tag{24}
\]

\[
D_t = (1 - \varepsilon)^{t-1}D_1 + \sum_{i \in I} \pi \varepsilon \frac{(1 - \eta_i)^t - (1 - \varepsilon)(1 - \varepsilon)^{t-1}}{\varepsilon - \eta_i} S_{t,1} + \sum_{i \in I} \sum_{\tau=1}^{t-1} a_i \pi \varepsilon \frac{(1 - \eta_i)^\tau - (1 - \varepsilon)^\tau}{\varepsilon - \eta_i} z_{t-\tau}, \tag{25}
\]

where \(S_{t,1}\) and \(D_1\) are taken as given at \(t = 1\), and then values for \(t > 1\) are defined by the expressions. If some climate change has taken place at the start of time \(t = 1\), we can write the system dependent on \(S_{t,1}, D_1 > 0\) — however, we can also rewrite the model to start at \(t = T\), possibly \(T < 0\), indicating the beginning of the industrial era, say 1850; we set \(z_t = 0\) for \(t < T\), and \(S_{t,T} = D_T = 0\). It is then immediate that the equation reduces to \(\square\). This defines the emissions-damage function \(\theta_{\tau}\) in Theorem 1. Q.E.D.

Lemma 3

We state first the following Lemma that will be used in other proofs and is also cited in the main text. The first item of Lemma 3 is an independence property following from the functional assumptions: the energy sector choices do not depend on the current state of the economy \((k_t, s_t)\). The latter item in Lemma 3 allows us to interpret the policy stringency as measured by \(h\) directly as the stringency of the carbon price \(\tau\).

Lemma 3 For all \(t\):

(i) Given policy sequence \((g_t, h_t)_{t \geq 0}\), emissions \(z_t = z_t^*\) at \(t\) implied by the policy are independent of the current state \((k_t, s_t)\), but depend only on the current technology at \(t\) as captured by \(A_t(.)\) and \(E_t(.)\);

(ii) Given the current state \((k_t, s_t)\) at \(t\), the carbon price, \(\tau_t = \partial y_t/\partial z_t\), satisfying \(\tau_t = h_t(1 - g_t)y_t\), is monotonic in the policy variable: \(d\tau_t/dh_t > 0\).

Proof: For given state and labour supply, \((k_t, s_t, l_t)\), output \(y_t = f_t(k_t, l_t, z_t, s_t)\) is increasing and concave in emissions \(z_t\), so that if the carbon price equals the marginal carbon product \(\tau_t = f_t,z = \partial y_t/\partial z_t\), we have \(dy_t/dz_t > 0\) and \(d\tau_t/dz_t < 0\). For a policy pair \((g_t, h_t)\) at time \(t\), we also derive \(dh_t/dz_t = [f_t,zz - (f_t,z)^2]/(1 - g)f_t < 0\), so that the carbon price measured in units \(h_t\) and the carbon price measured in units \(\tau_t\) are monotonically related, \(d\tau_t/dh_t > 0\).
The first-order conditions for fossil-fuel use $z_t$, and the labor allocations over the final goods $l_{y,t}$ and the energy sectors $l_{e,t}$ give:

$$\frac{1}{y_t} \frac{\partial y_t}{\partial e_t} \frac{\partial E_t}{\partial z_t} = h_t(1 - g_t), \quad (26)$$

$$\frac{\partial A_t}{\partial l_{y,t}} = \frac{\partial A_t}{\partial e_t} \frac{\partial E_t}{\partial l_{e,t}}, \quad (27)$$

Equation (27) balances the marginal product of labor in the final good sector with the indirect marginal product of labor in energy production. We have thus four equations, energy production (4), labour market clearance (5), and the two first-order conditions (26)-(27), that jointly determine four variables: $z_t, l_{y,t}, l_{e,t}, e_t$, only dependent on technology at time $t$ through $A_t(l_{y,t}, e_t)$ and $E_t(z_t, l_{e,t})$, but independent of the state variables $k_t$ and $s_t$. Thus, $z_t = z_t^*$ can be determined independently of $(k_t, s_t)$. Q.E.D.

**Proof of Theorem 2**

The proof is by induction. Induction hypothesis: assume (i) that future policies are given by a sequence of constants $(g_\tau, h_\tau)_{\tau > t}$ such that

$$k_{\tau + 1} = g_\tau y_\tau, \quad (28)$$

$$\frac{\partial y_\tau}{\partial z_\tau} = h_\tau(1 - g_\tau)y_\tau, \quad (29)$$

and (ii) that Theorem 2 holds for $t + 2$. We can thus construct the value function for the next period, as

$$W_{t+1}(k_{t+1}, s_{t+1}) = u_{t+1} + \delta W_{t+2}(k_{t+2}, s_{t+2}).$$

Consider policies at $t + 1$. From (28), $k_{t+2} = g_{t+1}y_{t+1}$. Emissions $z_{t+1} = z_{t+1}^*$ can be determined independently of the state variables $k_{t+1}$ and $s_{t+1}$ as shown in Lemma 3. Substituting the policies at $t + 1$ gives:

$$W_{t+1}(k_{t+1}, s_{t+1}) = [\ln(1 - g_{t+1}) + \ln(A_{t+1}) + \alpha \ln(k_{t+1}) + \ln(\omega(s_{t+1}))] - \Delta_u D_{t+1}$$

$$+ \delta \tilde{A}_{t+2} + \delta \xi [\ln(g_{t+1}) + \ln(A_{t+1}) + \alpha \ln(k_{t+1}) + \ln(\omega(s_{t+1}))] + \delta \Omega(s_{t+2})$$

Collecting the coefficients that only depend on future policies $g_\tau$ and $z_\tau$ for $\tau > t$, and that do not depend on the next-period state variables $k_{t+1}$ and $s_{t+1}$, we get the constant part of $V_{t+1}(k_{t+1})$:

$$\tilde{A}_{t+1} = \ln(1 - g_{t+1}) + \delta \xi \ln(g_{t+1}) + (1 + \delta \xi) \ln(A_{t+1}) - \delta \zeta_1 z_{t+1} + \delta \tilde{A}_{t+2}. \quad (30)$$
Collecting the coefficients in front of \( \ln(k_{t+1}) \) yields the part of \( V_{t+1}(k_{t+1}) \) depending \( k_{t+1} \) with the recursive determination of \( \xi \),

\[ \xi = \alpha(1 + \delta \xi). \]

so that \( \xi = \frac{\alpha}{1 - \alpha \delta} \) follows.

Collecting the terms with \( s_{t+1} \) yields \( \Omega(s_{t+1}) \) through

\[ \Omega(s_{t+1}) = \ln(\omega(s_{t+1}))(1 + \delta \xi) - \Delta_u D_{t+1} + \delta \Omega(s_{t+2}). \]

where \( z_{t+1} = z^*_{t+1} \) appearing in \( s_{t+2} = (z_1, \ldots, z_t, z_{t+1}) \) is independent of \( k_{t+1} \) and \( s_{t+1} \) so that we only need to consider the values for \( z_1, \ldots, z_t \) when evaluating \( \Omega(s_{t+1}) \). The values for \( \zeta_\tau \) can be calculated by collecting the terms in which \( z_{t+1-\tau} \) appear. Recall that \( \ln(\omega(s_{t+1})) = -D_{t+1} \) so that

\[ \zeta_\tau = ((1 + \delta \xi) + \Delta_u) \sum_{i \in I} a_i \pi \varepsilon \frac{(1 - \eta_i)^\tau - (1 - \varepsilon)^\tau}{\varepsilon - \eta_i} + \delta \zeta_{\tau+1} \]

Substitution of the recursive formula, for all subsequent \( \tau \), gives

\[ \zeta_\tau = \left( \frac{1}{1 - \alpha \delta} + \Delta_u \right) \sum_{i \in I} \sum_{t=\tau}^\infty a_i \pi \varepsilon \delta^{t-\tau} \frac{(1 - \eta_i)^t - (1 - \varepsilon)^t}{\varepsilon - \eta_i} \]

To derive the value of \( \zeta_1 \), we consider

\[ \sum_{t=1}^\infty \delta^{t-1} \frac{(1 - \eta_i)^t - (1 - \varepsilon)^t}{\varepsilon - \eta_i} \]

\[ = \sum_{t=1}^\infty [\delta(1 - \eta_i)]^t - \sum_{t=1}^\infty [\delta(1 - \varepsilon)]^t \]

\[ = \delta(1 - \eta_i) - \delta(1 - \varepsilon) \frac{\delta(1 - \eta_i) - \delta(1 - \varepsilon)}{1 - \delta(1 - \eta_i)} \]

\[ = \frac{1}{1 - \delta(1 - \eta_i)} \frac{1}{1 - \delta(1 - \varepsilon)} \]

(When \( \eta_i = \varepsilon \), \( \zeta_1 \) still has a closed-form solution; this derivation is available on request)

Q.E.D.

**Proof of Proposition 1**

In text.

**Proof of Proposition 2**

In text.
Proof of Proposition 3

To prove the result, set $\delta = \gamma^*$ and $\beta = 1$, and optimal policy for the Ramsey planner follows from Proposition 2. For such a planner, Theorem 2 defines the present-value future marginal utility losses from emissions through

$$\frac{\partial \Omega(s_{t+1})}{\partial z_t} = \Delta \gamma \sum_{i \in I} \gamma \pi a_i \varepsilon [1 - \gamma (1 - \eta_i)] [1 - \gamma (1 - \varepsilon)]$$

where $\gamma = \gamma^*$. Since the planner sets

$$u_t \frac{\partial y_t}{\partial z_t} = \gamma^* \frac{\partial \Omega(s_{t+1})}{\partial z_t},$$

the optimal policy has the interpretation given in Proposition 3. Q.E.D.

Proof of Proposition 4

Because of Lemma 3 (ii), the proposition, stated as $\frac{\tau^{\beta \delta}}{\tau_i} > 1$, can be rewritten, equivalently, as one where carbon prices are measured in utility units: $\frac{h^{\beta \delta}}{h_i} > 1$. We consider the latter ratio for very long climate change delays, $\eta_i = \varepsilon = 0$, and, $\beta < 1$:

$$\frac{h^{\beta \delta}}{h_i} = \frac{(1 - \gamma)^2 \beta \delta}{(1 - \delta)^2 \gamma} = \left(1 - \frac{1 - \beta \delta}{1 - \alpha \delta + \alpha \beta \delta}\right)^2 \left(1 - \alpha \delta + \alpha \beta \delta\right) = \frac{(1 - \delta (\alpha + (1 - \alpha) \beta))^2}{(1 - \delta)^2 (1 + \alpha \delta (\beta - 1))} > 1$$

The first equality follows from substitution of $\eta_i = \varepsilon = 0$ in the equation for the equilibrium carbon price and efficient carbon price. The second equality substitutes the value for $\gamma$. The final inequality follows as for $\beta < 1$, we have that $\alpha + (1 - \alpha) \beta < 1$, and thus the numerator exceeds $1 - \delta$, while $\beta < 1$ also ensures that the second term in the denominator falls short of 1. Q.E.D.

Proof of Proposition 5

From Proposition 2, $\tau^{\beta \delta} \to \infty$ as $\delta \to 1$. From Proposition 4, we see that $\tau^\gamma$ remains bounded. Q.E.D.
Proof of Lemma 2

Consider a given policy path \((g_τ, z_τ)_{τ≥t}\). We look at variations of policies at time \(τ\), and consider the effect on welfare at time \(t\). All effects are captured by \(W_{t+1}\) in Theorem 2. The analysis in the proof of Theorem 2 implies: the value function at time \(t\) is separable in states and the parameters \(ξ\) and \(ζ\) do not depend on future polices \((g_τ, z_τ)\), but term \(\tilde{A}_t\) does. Technically, we need to show that, for some given \(τ > t\),

1. \(\tilde{A}_t\) increases in \(g_τ\) for \(g_τ < αδ\),

2. \(\tilde{A}_t\) decreases in \(z_τ\) for \(z_τ > z_δ^t\),

where \(z_δ^t\) is the emission level that is consistent with the policy variable \(h^δ\) and \(z_τ\) is the emission level consistent with some \(h < h^δ\). In the proof of Theorem 2 consider (30). Term \(\tilde{A}_t\) increases with \(\tilde{A}_τ\) for some \(τ > t\). Moreover, \(\tilde{A}_τ\) is strictly concave in \(g_τ\), and maximal when \(g_τ\) maximizes \(\ln(1−g_τ)+δξ\ln(g_τ)\), that is, for \(g_τ = \frac{δξ}{1+δξ} = αδ\). This proves item 1. Furthermore, notice that \(A_τ\) depends on \(z_τ\), that \(\tilde{A}_τ\) is strictly concave in \(z_τ\) and maximal when \(z_τ\) maximizes \((1+δξ)\ln(A_τ(z_τ))−δζ_1z_τ\), that is, for \(\deltaζ_1=δζ_1(1−αδ)\). This is the value of \(z_τ\) consistent with \(h^δ\). This proves item 2. We have now shown the “if” part of Lemma 2. The “only if” follows from the strict concavity of \(\tilde{A}_τ\) with respect to \((g_τ, z_τ)\). Q.E.D.

Proof of Proposition 6

By Theorem 2, the change of the carbon price to the imputed Pigouvian price does not affect policy \(g\); thus, we can focus on the change in current welfare \(w_t\) due to the effect of future carbon prices. Also, by Lemma 3, a higher policy \(h\) implies a higher carbon price \(τ\). Let \(β < 1\) so that \(βδ < γ < δ\), and let climate change be a slow process such that \(τ^δ_1 > τ^{βδ} > τ^γ_1\); see Proposition 4. Imposing the imputed carbon price will then decrease the future carbon price, taking it further away from \(τ^δ_1\), decreasing current welfare as shown in Lemma 2. The same mechanism applies for \(β > 1\), when we have \(τ^δ_1 < τ^{βδ} < τ^γ_1\). Moreover, imposing the imputed carbon price on current policies implies a deviation from the current best response. That is, both changes induced, those in the present and future polices, decrease the present welfare.
Proof of Proposition 7

The proposal $\hat{g}_t$, as defined in the text, maximizes $w_t = u_t + \beta \delta W_{t+1}(k_{t+1}, s_{t+1})$. From the proof of Theorem 2, we see that $W_{t+1}(k_{t+1}, s_{t+1})$ depends on $g_{t+1}$ only through $\tilde{A}_{t+1}$ so that

$$\tilde{A}_{t+1} = \ln(1 - g_{t+1}) + \delta \xi \ln(g_{t+1}) + (1 + \delta \xi) \ln(A_{t+1}) - \delta \zeta_1 z_{t+1} + \delta \tilde{A}_{t+2}$$

$$\forall \tau > t, \hat{g}_\tau = g \Rightarrow \tilde{A}_{t+1} = \frac{1}{1 - \delta} \ln(1 - g) + \frac{\delta \xi}{1 - \delta} \ln(g) + (1 + \delta \xi) \sum_{\tau=t+1}^{\infty} \delta^{\tau-t-1} [\ln(A_\tau) - \delta \zeta_1 z_\tau]$$

$$\Rightarrow \arg \max_g w_t = \arg \max_g \ln((1 - g) y) + \beta \delta \tilde{A}_{t+1}$$

$$= \arg \max_g \ln(1 - g) + \beta \delta \xi \ln(g) + \frac{\beta \delta}{1 - \delta} \ln(1 - g) + \frac{\beta \delta^2}{1 - \delta} \xi \ln(g)$$

$$\Rightarrow \hat{g}_t = \tilde{g} = \frac{\alpha \beta \delta}{1 + \alpha \delta (\beta - 1) + (1 - \alpha \delta)(\beta - 1) \delta}.$$

Since $\tilde{g}$ it is independent of $t$, this same proposal is optimal for any agent at $\tau > t$. It remains to be shown that the policy is self-enforcing, so that for any $t$ there is no one-shot deviation $g_t \neq \hat{g}$ implying a higher payoff $w_t$. From Theorem 2, the one-shot deviation at $t$ is independent of the policy sequence $(g_\tau, h_\tau)_{\tau > t}$; that is, it is the Markov policy, $g_t = g^*$. Anticipating that $g_t = g^*$ triggers $(g_\tau = g^*, h_\tau = h^*)_{\tau > t}$, gives then the Markov payoff as the deviation payoff. But since, by construction of $\hat{g}$, $w_t^* < w_t$, where the latter is supported by the continuation policy, the deviation leads to a strict loss. (Since $w_t$ is separable in $(g, h)$, the argument holds independently of whether a deviation for one policy triggers the Markov response for the other policy.)

Consider then the proposal $\hat{h}_t$ that maximizes $w_t = u_t + \beta \delta W_{t+1}(k_{t+1}, s_{t+1})$. From Theorem 2, we see that $W_{t+1}(k_{t+1}, s_{t+1})$ depends on $h_\tau$ through $z_\tau$ for $\tau > t$; from Lemma 3, $z_t$ is a monotonic (strictly decreasing) function of $h_t$. Thus, from Theorem 2, $W_{t+1}(k_{t+1}, s_{t+1})$ depends on $z_\tau$ and $h_\tau$ for $\tau > t$ only through $\tilde{A}_{t+1}$,

$$\beta \delta (1 + \delta \xi) \sum_{\tau=t+1}^{\infty} \delta^{\tau-t-1} [\ln(A_\tau(z_\tau)) - \delta \zeta_1 z_\tau]$$
where we have written $A_r = A_r(z_r)$ to emphasize dependence on $z_r$. Hence,

$$\arg\max_h w_t = \arg\max_h \ln((1 - g)y) - \beta \delta \zeta_1 z_t + \beta \delta \bar{A}_{t+1}$$

$$= \arg\max_h \ln(A_t(z_t)) - \beta \delta \zeta_1 z_t + \beta \delta (1 + \delta \xi) \sum_{\tau = t+1}^{\infty} \delta^{\tau-t-1} [\ln(A_\tau(z_\tau)) - \delta \zeta_1 z_\tau].$$

Then, if $(\forall \tau \geq t, \hat{h}_t = h)$ is such a proposal at $t$, it solves

$$\left[ \frac{d \ln A_t(z_t)}{dz_t} - \beta \delta \zeta_1 \right] \frac{dz_t}{dh} = -\beta \delta (1 + \delta \xi) \sum_{\tau = t+1}^{\infty} \delta^{\tau-t-1} \left[ \frac{d \ln A_\tau(z_\tau)}{dz_\tau} - \delta \zeta_1 \right] \frac{dz_\tau}{dh},$$

where $dh = dz_t[f_{t,zz} - (f_{t,z})^2]/(1 - g)f_t$ from Lemma 3. The proposal is independent of $t$, that is, for $t \neq t', \hat{h}_t = \hat{h}_{t'} = h$ if and only if both $A_t = A_{t'}$ and $dz_t/dh = dz_{t'}/dh$. Thus, non-stationary technology $A_t \neq A_{t'}$ rules out a joint proposal $\hat{h}$ that is independent of time. Thus, Markov default follows, $\hat{h} = h^*$. Q.E.D.

Proof of Proposition 8

Follows directly from the expression for $\hat{\gamma}$. Q.E.D.

Calibrating carbon cycle

For calibration, we take data from Houghton (2003) and Boden et al. (2011) for carbon emissions in 1751–2008; the data and calibration is available in the supplementary material. We calibrate the model parameters $M, b, \mu$, to minimize the error between the atmospheric concentration prediction from the three-reservoir model and the Mauna Loa observations under the constraint that $CO_2$ stocks in the various reservoirs and flows between them should be consistent with scientific evidence as reported in Fig 7.3 from the IPCC fourth assessment report from Working Group I (Solomon et. al. 2007). There are 4 parameters to be calibrated. We set $b = (1, 0, 0)$ so that emissions enter the first reservoir (athmosphere). The matrix $M$ has 9 elements. The condition that the rows sum to one removes 3 parameters. We assume no diffusion between the biosphere and the deep ocean, removing 2 other parameters. We fix the steady state share of the deep ocean at 4 times the atmospheric share. This leaves us with 3 elements of $M$ to be calibrated, plus $\mu$. In words, we calibrate: (1) the $CO_2$ absorption capacity of the “atmosphere plus upper ocean”; (2) the $CO_2$ absorption capacity of the biomass reservoir relative to the atmosphere; and (3) the $CO_2$ absorption capacity of the deep ocean relative to the atmosphere. 

Follow the link [https://www.dropbox.com/sh/q9y9112j3l1ac6h/dgYpKVoCMg](https://www.dropbox.com/sh/q9y9112j3l1ac6h/dgYpKVoCMg)
atmosphere, while we fix the relative size of the deep ocean reservoir at 4 times the atmosphere, based on the IPCC special report on CCS, Fig 6.3 (Caldeira and Akai, 2005); (3) the speed of CO$_2$ exchange between the atmosphere and biomass, and (4) between the atmosphere and the deep ocean.

We transform this annual three-reservoir model into a decadal reservoir model by adjusting the exchange rates within a period between the reservoirs and the shares of emissions that enter the reservoirs within the period of emissions. Then, we transform the decadal three-reservoir model into the decadal three-box model, following linear algebra steps described above. The transformed box model has no direct physical meaning other than this: box 1 measures the amount of atmospheric carbon that never depreciates; box 2 contains the atmospheric carbon with a depreciation of about 7 per cent in a decade; while carbon in box 3 depreciates 50 per cent per decade. About 20 per cent of emissions enter either the upper ocean reservoir, biomass, or the deep ocean within the period of emissions. In the box representation, they do not enter the atmospheric carbon stock, so that the shares $a_i$ sum to 0.8. Our procedure provides an explicit mapping between the physical carbon cycle and the reduced-form model for atmospheric carbon with varying depreciation rates; the Excel file available as supplementary material contains these steps and allows easy experimentation with the model parameters. The resulting boxes, their emission shares, and depreciation factors are as reported in the text.

**Figure 1: calibrating damage-response functions**

For Figure 1, we calibrate our response function for damages, presented as a percentage drop of output, to those in Nordhaus (2007) and Golosov et al. (2014). The GAMS source code for the DICE2007 model provides a precise description of the carbon cycle through a three-reservoir model. We use the linear algebra from the previous Appendix to convert the DICE reservoir model into a three-box model, using Matlab (the code is available in the supplementary package). This gives the parametric representation of the DICE2007 carbon cycle through $a = (0.575, 0.395, 0.029)$, $\eta = (0.306, 0.034, 0)$. To find the two remaining parameters $\pi$ and $\varepsilon$ for calibrating our representation to DICE2007, we consider a series of scenarios presented in Nordhaus (2008), each with a different policy such as temperature stabilization, concentration stabilization, emission stabilization, the Kyoto protocol, a cost-benefit optimal scenario, and delay scenarios. For each of these!

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49As explained above, the decay rates in the final model come from the eigenvalues of the original model.
scenarios we calculated the damage response function by simulating an counterfactual scenario with equal emissions, apart from the first period when we decreased emissions by 1GtCO$_2$ (Gigaton rather than Teraton used in the tex to keep the impulse marginal for the purposes here). Comparison of the damages, relative of output, then defines the response function $\theta_\tau$ for that specific scenario. It turns out that the response functions are very close, and we take the average over all scenarios. Finally, we search for the values of $\pi$ and $\varepsilon$ that approximate the average response $\theta_\tau$ as closely as possible. We find $\varepsilon = 0.156$ [decade$^{-1}$], $\pi = 0.0122$ [TtCO$_2^{-1}$].

Golosov et al. is matched by setting $a = (0.2, 0.486)$, $\eta = (0, 0.206)$; they have no temperature delay structure, so that $\varepsilon = 1$. Figure 1 presents the emissions damage responses.
APPENDIX FOR ONLINE PUBLICATION
Appendix: A three-period extension to general functional forms

Technologies and preferences

Consider three generations, living in periods $t = 1, 2, 3$. In each period, consumers are represented by an aggregate agent having a concern also for future consumers’ utilities and welfare. Generations care about current and future utilities as follows

$$w_1 = u_1(c_1) + \beta [\delta u_2(c_2) + \delta^2 u_3(c_3)]$$ (31)
$$w_2 = u_2(c_2) + \beta [\delta u_3(c_3)]$$ (32)
$$w_3 = u_3(c_3),$$ (33)

where all utility functions $u_t$ are assumed to be continuous and, in addition, strictly concave, differentiable, and satisfying $\lim_{c \to 0} u_t' = \infty$. The condition $\beta < 1$ is equivalent to pure altruism towards future decision makers (Saez-Marti and Weibull 2005):

$$w_1 = u_1(c_1) + a_2 w_2 + a_3 w_3$$ (34)
$$a_2 = \beta \delta > 0, a_3 = \beta (1 - \beta) \delta^2 > 0,$$

where $a_2, a_3$ can be interpreted as welfare weights given by the first generation, implied by increasing patience over time. When $\beta = 1$, there is one-period pure altruism, and the typical recursive-dynastic representation of welfare follows.

In the first period, the consumption possibilities are determined by a strictly concave neoclassical production function $f_1(k_1, z)$, where $k_1$ is the capital stock, and $z$ is the use of fossil fuels, or emissions of carbon dioxide, both having positive marginal products, $\frac{\partial f_1}{\partial k} = f_{1,k}, \frac{\partial f_1}{\partial z} = f_{1,z} > 0$. The first generation starts with a capital stock $k_1$, and produces output using $z$, which can be used to consume $c_1$, or to invest in capital for the immediate next period $k_2$:

$$c_1 + k_2 = f_1(k_1, z).$$ (35)

We abstract from fossil-fuel use in the second and third period, but the first-period fossil-fuel use impacts production negatively in the third period: this captures the delay of climate-change impacts. The second agent starts with the capital stock $k_2$, produces output using a strictly concave neoclassical production function $f_2(k_2)$, and can use its income to consume $c_2$, or to invest in capital for the third period $k_3$:

$$c_2 + k_3 = f_2(k_2).$$ (36)
The third consumer derives utility from its consumption, which equals production. Past emissions now enter negatively, as damages, in the production function, $f_{3,k} > 0$, $f_{3,z} < 0$:

$$c_3 = f_3(k_3, z).$$  \hspace{1cm} (37)

We assume that also this production function is strictly concave.

An allocation $(c, k, z) = (c_1, c_2, c_3, k_2, k_3, z) \in A \subseteq \mathbb{R}_+^6$ (convex set) constitutes a consumption level for each generation $c_t$, the first-period use of fossil fuels $z$, which we thus also consider a proxy for the emissions of carbon dioxide emissions, and capital stocks $k_2$ and $k_3$ left for future agents ($k_1$ is given).

**Equilibrium carbon price**

In the subgame-perfect equilibrium generations choose consumptions and emissions in the order of their appearance in the time line, given the preference structure (31)-(33) and choice sets defined through (35)-(37).

The third agent consumes all capital received and cannot influence past emissions. The second agent decides on the capital $k_3$ transferred to the third agent, given the capital inherited $k_2$ and the emissions $z$ chosen by the first agent. We thus have a policy function $k_3 = g(k_2, z)$, defined by

$$\max_{k_3} u_2(c_2) + \beta \delta u_3(f_3(k_3)),$$  \hspace{1cm} (38)

leading to equilibrium condition

$$u'_2 = \beta \delta u'_3 f_{3,k} \Rightarrow 1 = \frac{R_{2,3}}{MRS_{2,3}},$$  \hspace{1cm} (39)

where we introduce the notation $R_{i,j}$ for the rate of return on capital from period $i$ to $j$, and $MRS_{i,j}$ for the absolute value of the marginal rate of substitution between consumptions in periods $i$ and $j$ for generation $t$.

The strict concavity of utility implies consumption smoothing, and thus if the second agent inherits marginally more capital $k_2$, the resulting increase in output is not saved fully but rather split between the second and third generation:

**Lemma 4** Policy function $g$ satisfies $0 < g_k < R_{1,2}$.

**Proof.** Substitute the policy function $k_3 = g(k_2, z)$ in (39),

$$\beta \delta u'_3(f_3(g(k_2, z), z)) f_{3,k}(g(k_2, z), z) = u'_2(f_2(k_2) - g(k_2, z)).$$  \hspace{1cm} (40)
Full derivatives with respect to $k_2$ lead to

$$\beta \delta g_k (u''_3 f_{3,k} f_{3,k} + u'_3 f_{3,k,k}) = u''_2 (f'_2 - g_k)$$

$$\Rightarrow g_k = \frac{f'_2 u''_2}{\beta \delta u''_3 f_{3,k} f_{3,k} + \beta \delta u'_3 f_{3,k,k} + u''_2} < f'_2 = R_{1,2}. \tag{41}$$
as $u''_1, f_{3,k,k} < 0$ and $f_{3,k}, u'_3 > 0$. ■

Understanding the second agent’s policy, the first agent decides on consumption and fossil-fuel use to maximize its welfare

$$w_1 = u_1 + \beta \delta [u_2 (f_2 (k_2) - g(k_2, z)) + \delta u_3 (f_3 (g(k_2, z), z))].$$

The choice for leaving capital $k_2$ satisfies

$$u'_1 = \beta \delta (f_{2,k} - g_k) u'_2 + \beta \delta^2 f_{3,k} g_k u'_3$$

$$\Rightarrow MRS^{t=1}_{1,2} = R_{1,2} + (\frac{1}{\beta} - 1) g_k. \tag{42}$$

where we use (39). When $\beta = 1$, preferences are consistent, and the term in brackets vanishes as in standard envelope arguments for single decision makers; capital $k$ is then valued according to the usual consumption-based asset pricing equation $MRS^{t=1}_{1,2} = R_{1,2}$. For $\beta < 1$, the second agent has a steeper indifference curve between consumptions in periods 2 and 3: the first-order effect in the bracketed term remains positive, leading to capital returns that no longer reflect the first generation’s consumption trade-offs.

Letting $MRS^{t=1}_{1,3} = MRS^{t=1}_{1,2} \times MRS^{t=1}_{2,3}$, we have

**Lemma 5** The compound capital return satisfies $MRS^{t=1}_{1,3} < R_{1,3}$ if and only if $\beta < 1$.

**Proof.** Using (42), $MRS^{t=1}_{2,3} = \beta MRS^{t=1}_{2,3} = \beta R_{2,3}$, and Lemma 4

$$MRS^{t=1}_{1,3} = MRS^{t=1}_{1,2} \times MRS^{t=1}_{2,3} = \left[ R_{1,2} + (\frac{1}{\beta} - 1) g_k \right] \times MRS^{t=2}_{2,3}$$

$$\Rightarrow MRS^{t=1}_{1,3} = \left[ R_{1,2} + (\frac{1}{\beta} - 1) g_k \right] \beta R_{2,3}$$

$$< \left[ R_{1,2} + (\frac{1}{\beta} - 1) R_{1,2} \right] \beta R_{2,3} = R_{1,3},$$

where the inequality holds iff $\beta < 1$. ■

Capital returns are generally excessive from the first agent’s point of view when $\beta < 1$, that is, the result holds without any restrictions on how emissions alter savings.
But, for the implications of the excessive capital returns on carbon pricing, we must make assumptions on the effect of first-period emissions on the second-period policy, \( g_z \).

Taking the full derivatives of (40) with respect to \( z \), we get

\[
g_z = -\frac{\beta(u''_3 f_{3,k} f_{3,z} + u'_{3,k} f_{3,kz})}{u''_2 + \beta u''_{3,k} f_{3,k} + \beta u'_{3,kk}}.
\] (43)

First note that all terms in the denominator are negative, so that with the overall negative sign in front, the signs of the numerator’s elements inform us about the mechanisms in play. The first term in the numerator captures the income effect of emissions and is positive. If the first generation emits more, the third generation has lower utility levels and the second generation will tend to save more, as the marginal utility of the third generation increases. The second term in the numerator captures the productivity effect and is negative. If the first generation emits more, productivity of capital in the third period will fall, and the return to investments in the second period will fall alongside. The relative strength of both mechanisms depends on the elasticity of marginal utility versus the elasticity of marginal damages:

\[
g_z > 0 \text{ iff } EMU > EMD
\] (44)

where \( EMU = -c_3 u''_3 / u'_3 \) is the elasticity of marginal utility, and \( EMD = f_3 f_{3,kz} / f_{3,k} f_{3,z} \) is the elasticity of marginal damages, and we use \( c_3 = f_3 \). If utility is more concave, then the left-hand side of the last inequality will increase, and the second generation will tend to save more with higher past emissions. If marginal damages increase more than proportionally with income, the right-hand side will increase and the second generation will tend to save less with higher emissions.

Assuming log utility, and that the production damage is multiplicative:

\[
\begin{align*}
u_t(c_t) &= \ln(c_t) \\
f_3(k_3, z) &= f_3(k_3) \omega(z),
\end{align*}
\] (45) (46)

where \( \omega(z) \) is a strictly decreasing damage function, sets both sides of the inequality to unity, and implies that the direct effect of emissions on savings vanishes, \( g_z = 0 \), as can be easily verified from (43).

**Lemma 6** The second generation does not adjust its savings to past emissions if utility is logarithmic and damages are proportional to output. More elastic marginal utility (or damages that increase less than proportionally with output) imply that second generation’s savings increase with past emissions.
Consider then the first generation’s equilibrium carbon policy \( z \):

\[
u'_1 f_{1,z} = \beta \delta g_z u'_2 - \beta \delta^2 (f_{3,k} g_z + f_{3,z}) u'_3. \tag{47}
\]

which after substitution of (39) can be rewritten as

\[
u'_1 f_{1,z} = (1 - (1 - \beta) g_z) \beta \delta^2 f_{3,k} u'_3 \tag{48}
\]
or

\[
MCP = (1 - (1 - \beta) g_z) \frac{MCD}{MRS_{1,3}^{z=1}} \tag{49}
\]

where we let \( MCP = f_{1,z} \) denote the marginal carbon product, and \( MCD = -f_{3,z} \) denote the marginal carbon damages. If \( \beta = 1 \), then capital returns reflect consumption trade-offs, \( MRS_{1,3}^{z=1} = R_{1,3} \), so that from (49) the carbon price becomes just equal to the damage, discounted with capital return:

\[
MCP = \frac{MCD}{R_{1,3}}. \tag{50}
\]

This is the general-equilibrium Pigouvian carbon price, under consistent preferences \( \beta = 1 \). If we impose (45)-(46) and thus \( g_z = 0 \), the first term in the carbon policy implied by (47) is unity. Yet, if \( \beta \neq 1 \), in equilibrium, while (49) continues to hold as an internal cost-benefit rule for \( t = 1 \), Lemma 5 implies that the discounted damage no longer equals the carbon price (if \( g_z = 0 \)):

\[
MCP > \frac{MCD}{R_{1,3}} \text{ if and only if } \beta < 1. \tag{51}
\]

In equilibrium, the first agent establishes a higher carbon price, compared to the Pigouvian level, if and only if \( \beta < 1 \), i.e., when the first agent gives a higher weight to the long-term utility than the second agent. The result has a very simple intuition. The first consumer would like to transfer more wealth to the third consumer, compared with the preferred wealth transfer of the second consumer: the high capital returns reflect this distortion (Lemma 5). The higher capital returns depress the present-value damages below the true valuation by the first consumer. The opposite deviation — carbon price below the Pigouvian price — occurs if \( \beta > 1 \).

**Proposition 9** Assume (45)-(46). If \( g_z = 0 \) but \( \beta \neq 1 \), the first-period carbon price does not satisfy the Pigouvian pricing rule, i.e., \( MCP \neq \frac{MCD}{R_{1,3}} \). The carbon price exceeds the Pigouvian level if and only if \( \beta < 1 \). Furthermore, for \( g_z \neq 1 \) and \( \beta < 1 \), we find that a larger elasticity of marginal utility with respect to consumption tends to lower
carbon prices while a larger elasticity of marginal damages with respect to income tends to increase carbon prices.

Proof. Above. □