# On the exhaustible-resource monopsony

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**Abstract**

The exhaustible-resource monopsony problem provides a basis for understanding the dynamic relationship between resource importers and suppliers. We find that the mere presence of a substitute supply creates a time-inconsistency problem for the monopsonist. When the buyer can commit to delaying the arrival of the substitute, he obtains a substantial reduction in resource prices but not enough as to appropriate the entire resource rent. In the absence of commitment, the equilibrium exhibits Coasian dynamics with sellers typically capturing a larger share of the rent; paradoxically, however, the sellers' surplus vanishes when the price of the substitute approaches infinity. The notion of the substitute can be mapped into the well-known connection between the durable-good monopoly and the exhaustible-resource monopsony.

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# 1 Introduction

There has long been an interest in understanding how a single buyer of an exhaustible resource —e.g., policy-maker interested in maximizing consumer surplus— might extract rents from (competitive) resource suppliers. Bergstrom (1982) first showed how an import tariff could allow the importer to extract the entire resource rent. Later, Maskin and Newbery (1990) and Karp and Newbery (1993) noticed that under more general conditions (e.g., increasing extraction costs) the resource importer suffers from a time-inconsistency problem that prevents him to extract the entire rent. They explain that if the importer can commit to tariffs, he would commit to higher tariffs in the future so as to force sellers to supply today at lower prices. In the absence of commitment, however, such threats are not necessarily credible, much the same way the durable-good monopolist may fail to commit to high prices in the future, as conjecture by Coase (1972). Indeed, Hörner and Kamien (2004) show that the two problems —durable-good monopoly and exhaustible-resource monopsony— are mirror images of each other, i.e., one problem can be obtained from the other by renaming variables.

In this paper, we revisit the exhaustible-resource monopsony problem but paying explicit attention to the fact that at the exhaustion of the resource stock the monopsonist switches to a substitute good that is in perfectly elastic supply. While it is central in resource economics that the price of the resource depends on the price of the substitute and the timing of its adoption (e.g., Nordhaus, 1973; Dasgupta and Heal, 1979), the time-inconsistency problem arising solely from the switch to the substitute and its implications for the equilibrium price path have not been noticed in the literature.[1] Given the importance of this feature in resource exhaustion, it is also of interest to see how the substitute maps into the connection identified by Hörner and Kamien (2004).

We find that under full commitment the resource buyer delays the switch to the substitute in order to postpone the arrival of the final price of the resource and, thereby, depress the price at which the resource is supplied today (in some cases the buyer may

[1] The issue does not arise in Maskin and Newbery (1990), for example, because they let the price of the subsitute, which is known to arrive at some point in the future, say at T, be lower than the (constant) cost of extracting oil. A time inconsistency problem still arises in their model because they add a fringe of competitive buyers that, from the perspective of the strategic buyer, brings heterogeneity to the supply side; otherwise the strategic buyer would face no commitment problems and would capture the entire rent as in Bergstrom (1982). Had the substitute been costlier than extracting oil, as in most resource models, it is not evident how the buyer in Maskin and Newbery (1990) could have resisted not buying the remaining oil at T together with the substitute and at the same (or slighly lower) price.

even commit to zero consumption during an interval of time in the future). Notice that despite full commitment the buyer fails to appropriate the entire resource rent; some surplus is lost due to the sellers' ability to compete with the substitute. The buyer's commitment solution is not time-consistent, however: delaying the arrival of the substitute leads to a price discount on all future consumption, so as the remaining resource stock is depleted the buyer has an incentive to move the switch to the substitute sooner than what was originally announced.

In the absence of commitment, the equilibrium exhibits Coasian dynamics with sellers typically capturing a larger fraction of the resource rent compared to that under commitment. Paradoxically, the sellers' surplus vanishes when the price of the substitute approaches infinity. The intuition is simple: when the substitute becomes extremely costly, the buyer can commit to postpone its arrival indefinitely, which destroys the sellers' ability to compete (and supply together) with it.

These results can be interpreted using the insights from the durable-goods monopoly problem. Kahn (1986) shows that the durable-goods monopolist can capture part of the surplus when there is production smoothing (coming from strictly convex costs) and that he can take all of it if consumers exhibit no heterogeneity in valuations (recall that such heterogeneity is necessary for the Coasian dynamics to arise in the durable-goods problem). In the resource model, on the other hand, the monopsonist can capture part of the surplus when there is consumption smoothing (coming from a strictly concave utility) and he can take all of it if resource suppliers exhibit no heterogeneity in costs (Hörner and Kamien, 2004). The substitute enters this picture by adding a higher-cost reproducible supply to the seller-side of the resource market, i.e., by introducing (or altering the existing) heterogeneity on the seller-side. However, the equilibrium timing of the switch to the substitute cannot be seen from the existing durable-goods models, simply because the notion of the substitute, or its analogue, is not present there. After solving the resource model (with the substitute), we explain how the solution can be mapped into the durable-goods framework by assuming a demand with durable and non-durable segments.

The results of the paper have implications for when and how demand-side policies can influence resource rents. In the context of climate change, for example, broad policy instruments that put a price on carbon emissions reduce the demand for fossil-fuel resources, but since they do not prevent the resource sellers from competing with the substitute technologies (e.g., renewable energies), they are distortionary and fall short in extracting rents. For the latter we require of more targeted instruments that can discriminate between resource and substitute suppliers such as tariffs on resource imports or subsidies on substitute technologies, in the spirit of discriminating price schedules for durable-goods (Bagnoli et al., 1989). Interestingly, the durable-good theory can provide insights on the optimal instrument design.[2]

Buyer power in resource markets can also arise in a situation where countries have resource endowments of different sizes in relation to their domestic demands. Besides stocks of crude oil, we can also think of other resources such as stocks of carbon quotas as part of a global market design where scarcity increases over time. A large buyer with a relatively small stock, like the US, must rely on supplies from countries with abundant stocks (i.e., endowments above their domestic needs). The large buyer cannot discriminate between the resource and substitute suppliers because of the remaining buyers in the market. This is another example of substitute suppliers influencing the Coasian dynamics (Liski and Montero, 2011).

# 2 Commitment solution

We assume that there is a single importer (buyer) of an exhaustible resource. The buyer's utility depends on the rate of consumption $q_t$, where time $t$ is a continuous variable. Utility from consumption is given by a strictly concave, increasing, and differentiable function $U(q_t)$. We assume that $U'(0) > \bar{p}$ where $\bar{p} > 0$ is the price of a (inexhaustible) substitute good (note that the substitute is available at all times). In the absence of other supply, the demand for this good is $q = \bar{q}$, with $\bar{p} = U'(\bar{q})$, which generates a surplus flow of $W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$.

There is an initial resource stock $Q_0$ in the hands of a large number of suppliers. Each resource supplier has one unit of the resource and a given cost of extracting and selling that unit. We assume a continuum of suppliers indexed by $Q \in [0, Q_0]$ and that the unit cost depends on this index. The unit cost is given by a non-increasing and differentiable function $c(Q)$. Let $\delta$ denote the discount rate, and let $p_t$ be the market price at which the resource can be sold at time $t$. A seller with cost $c(Q_t)$ is indifferent between selling at $t$ or after $\Delta$ units of time when $p_t - c(Q_t) = e^{-\delta\Delta}(p_{t+\Delta} - c(Q_t))$. As $\Delta \to 0$, this indifference becomes

[2] Building upon the theory of optimal commodity taxation —where the government has to allocate the burden of taxation across different sectors and can commit to tax schedules— Daubanes and Lasserre (2011) show that exhaustible resources should be taxed in priority following the standard inverse-elasticity (Ramsey) rule.

$$\frac{dp_t}{dt} = \delta(p_t - c(Q_t)),$$

which is the Hotelling rule. Obviously, the rule applies only when $p_t < \bar{p}$, so that in order to create a market for the resource we assume that the substitute price $\bar{p}$ strictly larger than the unit cost of the last resource unit $c(0)$.

The commitment solution is the consumption path $(q_t)_{t\geqslant 0}$ that maximizes the buyer's surplus in present value. The buyer announces the path at time $t = 0$ taking into account the resource constraint, the availability of the substitute, and the price arbitrage dictated by the Hotelling rule (as long as sellers are holding some strictly positive stock). The buyer's problem can be written as the one where the choice is made over the resource consumption path $(q_t)_{t\geqslant 0}$ as well as end of the path $T$ which also marks the switch to the substitute:

$$V_{t=0} = \max_{\{q_t,T\}} \int_0^T \{U(q_t) - p_t q_t\} e^{-\delta t} dt + \frac{1}{\delta} e^{-\delta T} W(\bar{q}) \tag{1}$$

$$\frac{dQ_t}{dt} = -q_t, Q_0 > 0,\ Q_T = 0, \tag{2}$$

$$\frac{dp_t}{dt} = \delta(p_t - c(Q_t)),\ p_T = \bar{p} > 0, \tag{3}$$

We impose the constraints that the final price equals the substitute price, $p_T = \bar{p}$, and that the resource stock is fully depleted, $Q_T = 0$. These boundary conditions follow from the assumption that the buyer cannot prevent resource sellers from supplying at the substitute price.

To illuminate, suppose $c(Q) = 0$ for all suppliers, and that the buyer's commitment solution involves leaving a minuscule $\varepsilon$-fraction of the stock in the ground and then switching to the substitute. This would force sellers to race for early sales, so prices at $t = 0$ and later would collapse to 0. It is immediately clear that the commitment solution cannot achieve this full rent extraction if the buyer cannot prevent the resource sellers from selling at $\bar{p}$ at the time he starts purchasing from substitute suppliers (the buyer has no means to price discriminate between resource and substitute suppliers). The price will jump to $\bar{p}$ right after the depletion of the stock and thus sellers will see $\bar{p}$ as the final price of the resource rather than zero.[3] The commitment solution must respect the stated restriction on the final price.

The buyer's optimal plan can be obtained using control theory (see, e.g., Karp 1984). Over the interval of time where resource sales are positive, the optimal consumption must satisfy the first-order condition

[3]The same reasoning applies for positive costs $c(Q) < \bar{p}$.

$$c(Q_t) - c'(Q_t)(Q_0 - Q_t) = U'(q_t) - \frac{1}{\delta}U''(q_t)\frac{dq_t}{dt}. \quad (4)$$

The left-hand side of (4) is the marginal cost from buying an extra unit, and the right-hand side is the marginal benefit of consuming that unit today rather than tomorrow.

Equation (4) is important as it identifies the time-inconsistency problem coming from heterogeneity in resource extraction costs. The buyer's optimal consumption plan depends on the initial stock $Q_0$ (i.e., the stock at the time of the initial announcement) because the monopsonist's inframarginal costs from additional purchases depend on total remaining consumption. If the monopsonist is allowed to revise his initial plan at some future date $t > 0$, the relevant "initial" stock at the time of this new announcement is not longer $Q_0$ but $Q_{t>0} < Q_0$, which implies the monopsonist would now like to consume faster than initially announced. This source of time-inconsistency has been carefully discussed in Karp and Newbery (1993). And Hörner and Kamien (2004) connect this time-inconsistency to the durable-goods model where the supplier's cost is replaced by the consumer's valuation.

Since our focus is the time-inconsistency problem coming from the substitute alone, in what follows we abstract from resource cost heterogeneity and set $c(Q) = c \geqslant 0$. Consider then the monopsonist's optimal time $T$ of switching to the substitute. The optimal choice should balance the marginal increase in the total payoff when resource consumption period is marginally prolonged against the cost from postponing the arrival of the substitute surplus $W(\bar{q})$. In Appendix A we derive the optimality conditions that combined yield the following condition for the buyer's optimal stopping time

$$W(\bar{q}) - \{U(q_T) - U'(q_T)q_T\} = \delta(\bar{p} - c)Q_0 > 0. \quad (5)$$

The two conditions, (4) and (5), fully pin down the buyer's commitment path.

Condition (5) carries the main economic insight of the commitment solution as it captures the time-inconsistency problem coming from the substitute. While the resource price at the time of its exhaustion is $p_T = \bar{p}$, the buyer's consumption at that time is only $q_T < \bar{q}$, i.e., the buyer commits to reduce consumption below the efficient level to the very end of the resource consumption path. This can be seen from the left-hand side of (5) which measures the deviation from the first-best surplus flow $W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$ at time $T$. According to (5), this deviation must be positive, which requires $q_T < \bar{q}$.[4] This

[4]Since $q_T \geq 0$, we cannot rule out the corner $q_T = 0$ if $W(\bar{q})$ is too small relative to $Q_0$. In such a case, the monopsonist commits to a consumption path that exhibits three stages: An initial stage where

also makes intuitive sense. Efficiency requires "smooth landing" of consumption, namely, $q_T = \bar{q}$, and by distorting consumption downwards the buyer balances out efficiency and rent extraction.

Delaying consumption is costly to the buyer, but the right-hand side of (5) captures the offsetting gain of doing so: it achieves a downward adjustment in the price path. In fact, the right-hand side of (5) is the marginal discount on the purchasing cost of the initial stock $Q_0$ when the length of the (resource) consumption path ($T$) is extended by one marginal unit of time. To see this, note that the initial price is $p_0 = c + (\bar{p} - c)e^{-\delta T}$, so a marginal delay in $T$ leads to a drop $\delta(\bar{p} - c)$ in the initial price $p_0$.

For the time-inconsistency problem, note that the downward distortion in the consumption path depends on the initial stock $Q_0$, which implies that the buyer would like to reconsider his original plan if allowed to do so at some later date $t > 0$. For example, near exhaustion the remaining stock is close to zero; hence, the buyer would like to bring $q_T$ much closer to $\bar{q}$. Since the buyer has already enjoyed the price reduction for most of the stock, he would now like to eliminate the consumption distortion that was used to obtain such price reduction.

# 3 Resource substitute and the Coasian dynamics

We now move to the analysis of equilibrium when the resource monopsonist cannot commit. To simplify the exposition, we set the extraction cost equal to zero, i.e., $c = 0$. In Appendix B we present the extensive form of the game for a discrete number of stages where at each stage the buyer first announces his demand and then the market prices the resource based on their (correct) expectations of future play. We show that the payoff-relevant history $h_t$ at each date $t$ is summarized by the remaining stock so that the buyer consumption rule and the equilibrium price depend only on the remaining stock, that is

$$q_t = C_t(Q_t) \text{ and } p_t = P_t(Q_t). \tag{6}$$

We adopt next a continuous-time formulation to characterize these functions.[5]

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the stock $Q_0$ is consumed except for an $\varepsilon$-fraction of it (otherwise, the last supplier in this first stage receives $\bar{p}$), followed by an interval of time of no consumption at all, and then by the final long-run stage with a flow consumption of $\bar{q}$ from substitute suppliers and the remaining $\varepsilon$-fraction of resource suppliers.

[5] As we explain in Appendix B, there is nothing in the structure of the game that prevents the continuous time formulation from capturing the limit of the discrete time setting.

## 3.1 Equilibrium solution

We look for a price equilibrium function $P(Q) : \mathbb{R}_+ \to [0, \bar{p}]$ with the following properties: (i) continuous and differentiable almost everywhere, (ii) stationary $P_t(Q_t) = P(Q_t)$, and (iii) decreasing from $P(0) = \bar{p}$. We will construct such a price function from the equilibrium conditions, as well as a stationary consumption rule $q_t = C(Q_t)$ for the buyer.[6] For a pair $(P(Q_t), C(Q_t))$, we can write the buyer's payoff as

$$V(Q_t) = \int_t^{\infty} [U(q_\tau) - P(Q_\tau)q_\tau]e^{-\delta(\tau - t)}d\tau \tag{7}$$

where $q_\tau = C(Q_\tau)$ and $\bar{q} = C(0)$ so that the surplus flow is $W(\bar{q})$ when the stock is zero. Over a small interval of time $\Delta$, we can approximate this value as

$$V(Q_{t-\Delta}) = [U(q_t) - P(Q_{t-\Delta} - q_t\Delta)q_t]\Delta + e^{-\delta\Delta}V(Q_{t-\Delta} - q_t\Delta). \tag{8}$$

where $Q_{t-\Delta}$ is the stock at the beginning of period $t$ (see the discrete time formulation in Appendix B) and $Q_t = Q_{t-\Delta} - q_t\Delta$ is the stock at the end of period $t$ or beginning of period $t + \Delta$. The equilibrium consumption $q_t$ for a given $Q_{t-\Delta}$, by definition, maximizes the right-hand side of (8). The buyer's payoff is generated by differentiable functions, so we can describe the equilibrium choice by differential methods. Maximizing (8) with respect to $q_t$ yields

$$[U'(q_t) - P(Q_t) + P'(Q_t)q_t\Delta]\Delta - e^{-\delta\Delta}V'(Q_t)\Delta = 0$$

and letting $\Delta \to 0$ gives (the direct price effect $P'(Q_t)q_t\Delta^2$ vanishes as $\Delta \to 0$)

$$U'(q_t) - P(Q_t) - V'(Q_t) = 0. \tag{9}$$

To evaluate now the opportunity cost of current consumption, i.e. $V'(Q_t)$, we take the total differential of $V(Q_t)$ in (7) with respect to $Q_t$

$$\frac{dV(Q_t)}{dQ_t} = \int_t^{\infty} [U'(q_\tau) - P(Q_\tau)][\frac{dq_\tau}{dQ_t}]e^{-\delta(\tau - t)}d\tau + \tag{10}$$

$$\int_t^{\infty} [-P'(Q_\tau)q_\tau][\frac{dQ_\tau}{dQ_t}]e^{-\delta(\tau - t)}d\tau \tag{11}$$

[6] In Appendix B, we state a condition for the extensive form implying that resource consumption stops before the end of the game, so the strategies become independent of the total time available for the strategic interaction. Under this condition, it is not surprising that in the limiting equilibrium where period length vanishes, a stationary equilibrium can be constructed.

The first line (10) collects terms that are affected by the perturbations of the equilibrium choice $(dq_\tau)_{\tau\geqslant t}$. But since we are evaluating these (marginal) perturbation along the equilibrium consumption path, i.e., $q_\tau = C(Q_\tau)$ for all $\tau \geq t$, line (10) must be zero, otherwise the buyer could profitably deviate from function $C(Q_\tau)$.

For the second line (11), the Hotelling rule implies $P(Q_t) = e^{-\delta(\tau-t)}P(Q_\tau)$ for $Q_\tau > 0$, from which we obtain the differential

$$\frac{dQ_\tau}{dQ_t} = \frac{P'(Q_t)}{P'(Q_\tau)}e^{\delta(\tau-t)} \tag{12}$$

Recalling that $T$ denotes the time of exhaustion of the stock and that $P'(0) = 0$, we can combine (12) with (10)-(11) to obtain

$$V'(Q_t) = -P'(Q_t)\int_t^T q_\tau d\tau = -P'(Q_t)Q_t, \tag{13}$$

From (9) and (13), we finally get that equilibrium consumption $q_t$ satisfies

$$U'(q_t) - P(Q_t) + P'(Q_t)Q_t = 0 \tag{14}$$

for all $t$.

The monopsonist follows the same general principle as its static analog: the marginal utility from consumption equals the marginal purchasing cost. However, the price function $P(Q_t)$ is forward-looking in that the marginal effect of current consumption on today's price depends on the overall remaining consumption $Q_t$.

We have verified that the buyer's best-response to a price function satisfies (14), and we will next verify that there exists a price function with the stated properties that solves (14). From the Hotelling rule for $Q_t > 0$ we have

$$P'(Q_t)\frac{dQ_t}{dt} = \delta P(Q_t)$$

or

$$q_t = -\frac{\delta P(Q_t)}{P'(Q_t)} \tag{15}$$

that combined with (14) gives

$$U'\left(-\frac{\delta P(Q)}{P'(Q)}\right) - P(Q) + P'(Q)Q = 0 \tag{16}$$

This is a first-order ordinary differential equation for $P(Q) \geq 0$, where $Q \in [0, \infty)$. The solution is a function that declines from the boundary condition $P(0) = \bar{p}$ and approaches zero as $Q$ becomes infinitely large.

It is difficult to convey further intuition from (16) without explicitly solving it. Hence, let us consider $U(q) = \ln(q)$, which implies that condition (16) becomes

$$P'(Q) = -\frac{\delta P(Q)^2}{1 - \delta P(Q)Q}$$

and has the solution[7]

$$P(Q) = \bar{p}\sqrt{\delta^2 \bar{p}^2 Q^2 + 1} - \delta \bar{p}^2 Q. \tag{17}$$

This equilibrium price function exhibits Coasian dynamics. First, and perhaps initially surprising, when the substitute price $P(0) = \bar{p}$ approaches infinity, the resource price at any strictly positive stock level $Q > 0$ approaches zero, i.e., $\lim_{\bar{p}\to\infty} P(Q)|_{Q>0} = 0$. In this limit, the equilibrium solution converges to the commitment outcome where the monopsonist commits to leave a minuscule fraction $\varepsilon$ of the stock in the ground that makes prices collapse to zero, and thereby, allows him appropriate the entire resource rent. The intuition is simple: when the substitute becomes extremely costly, the buyer can commit to postpone its arrival indefinitely, which destroys the sellers' ability to compete with the substitute. Second, as the interest rate $\delta$ falls to zero, the price function becomes almost flat at the choke level, i.e., $P(Q) = \bar{p}$ for all $Q > 0$. The buyer then suffers from the Coase conjecture: the entire resource surplus goes to the seller side.[8] Outside these limiting cases, suppliers and monopsonist share the resource surplus.

Although it cannot be seen directly from the equilibrium function (17), the monopsonist can also capture the entire resource rent when his choke price is below the substitute price, i.e., $U'(0) \leqslant \bar{p}$; in other words, when the substitute is worthless to him ($W(\bar{q}) = 0$).

**Proposition 1** *Consider a given resource stock $Q_t$ to be consumed by a monopsonist at an arbitrarily frequently rate. The full resource surplus goes to the buyer if (i) the price of the substitute is arbitrarily large, or (ii) the substitute has no value to the buyer. The full resource surplus goes to the sellers when discounting vanishes. With discounting and a valuable substitute, the surplus is shared between the buyer and the competitive suppliers.*

***Proof.*** *See above.* ■

[7] In solving the equation, we took advantage of the one-to-one relationship between $P$ and $Q$ and solved instead for $Q(P)$. Inverting that solution gives equation (17).

[8] However, one may argue that highly frequent trading is the limit where the conjecture should be tested both in durable goods and exhaustible resources, and there is no reason to consider zero discounting *per se*. In durable goods, frequent trading translates into a short time to be served, meaning a low effective discounting. This is the meaning of patience in durable goods, and not zero discounting.

With the aid of (17) we can also visualize how equilibrium prices and consumption evolve over time. In Figure 1, the thick solid line is the equilibrium price path $p_t = p_0 e^{\delta t}$, where $p_0 = \bar{p}\sqrt{\delta^2 \bar{p}^2 Q_0^2 + 1} - \delta \bar{p}^2 Q_0$, while the thin solid line is the perfectly competitive price path $p_t^* = p_0^* e^{\delta t}$, where $p_0^* = 1/(1+\delta Q_0)$. This latter path also represents the socially optimal marginal utility path. Interestingly, the equilibrium marginal utility $u'(q_t)$ — the broken line— follows closely the socially optimal price when the stock is relatively large, suggesting that the monopsony seeks to avoid large consumption distortions when scarcity is still low. This is turn indicates that the price discount is mostly achieved by consuming near (and above) the long-run level $\bar{q}$ for an extended period of time. The reason why the buyer can credibly postpone the switch to the substitute when approaching the exhaustion of the resource is the proximity to the competitive allocation where the buyer's surplus loss from consumption distortion is small.

*** INSERT FIGURE 1 HERE OR BELOW ***

It is natural to ask now whether and to what extent the above logic changes when extraction costs depend on the remaining stock, i.e., when unit cost increases with depletion, $c'(Q_t) < 0$. Using the boundary for the price path and the Hotelling rule, we can express the equilibrium price as

$$p_t = e^{-\delta(T-t)}\bar{p} + \int_t^T \delta c(Q_\tau) e^{-\delta(\tau - t)} d\tau.$$

The resource cannot be sold for anything less than $\bar{p}$ and, therefore, the equilibrium price converges to this level independently of whether the stock is economically (last units not extracted, $\bar{p} < c(0)$) or physically depleted (all units extracted, $\bar{p} > c(0)$). The equilibrium delay in buyer's consumption lowers the price path by postponing the arrival of the substitute much the same way as explained above —the substitute price appears independently of the cost structure in the price equation. This effect is a source of the above delay dynamics, as long as the buyer is switching to the substitute at some point in the future (i.e., $W(\bar{q}) > 0$). For these reasons, Proposition 1 applies under more general cost structures.

### 3.2 Connections to the durable-good monopoly

We are now in a position to connect these results to the literature on the Coase conjecture.[9] First, the buyer can implement his first-best if he can separate the resource sellers from the substitute suppliers. This would be the case if, for example, there were a single importer of oil capable of producing the substitute himself and setting a tariff on oil imports. From the resource sellers' point view, this situation is not different than assuming $W(\bar{q}) = 0$; in either case there is no market after the depletion of the stock. This corresponds to the solution suggested by Bagnoli et al. (1989) where the Coase conjecture is avoided by a discriminating price schedule.[10]

Second, in the absence of the substitute our setting coincides with that in Kahn (1986) and thus also with Hörner and Kamien (2004) who build the connection between the two problems. To export the notion of the substitute into the Kahn's framework, one could consider two groups of consumers as follows. The first group of consumers includes a continuum of agents indexed by $S \in [0, \bar{S}]$ with valuations represented by function $P(S)$ depending on the index; the valuations go from $P(0)$ to $P(\bar{S}) \leq P(0)$. The second group is an almost infinite measure of identical consumers each with positive valuation $\underline{p} < P(\bar{S})$ for the durable. If $\gamma(q)$ is the monopolist's strictly convex production cost, his long-term surplus is $W(\underline{q}) = \underline{pq} - \gamma(\underline{q}) > 0$, where $\gamma'(\underline{q}) = \underline{p}$. Provided the monopolist cannot price discriminate between the two group of consumers, he would suffer from the same time-inconsistency problems than our resource monopsonist, even if $P(\bar{S}) = P(0)$. Our conjecture is that the solution to this durable-good problem would be a mirror-image of the resource monopsony solution, as concluded by Hörner and Kamien (2004) for the model without the substitute.

Finally, we note some key differences between our equilibrium with a substitute and the one obtained in Kahn. As discounting vanishes, the monopolist in Kahn can almost perfectly price discriminate among consumers (pp. 287-288). The exact opposite, however, occurs in the resource model with a substitute: competitive agents (i.e., sellers)

[9]The conjecture was presented in Coase (1972). In the literature that follows, the conjecture is understood as the entire loss of monopoly power when consumers are patient enough. Early formalizations are Stokey (1981), and Bulow (1982). The monopolist may escape the conjecture, at least partially, if: marginal production costs are convex (Kahn, 1986); reputational strategies can be used (Ausubel and Deneckere, 1989); a price-quantity scheme can be used to discriminate among discrete buyers (Bagnoli et al., 1989); the good depreciates (Karp, 1996); there is entry of new consumers (Sobel, 1991); or there are capacity costs (McAfee and Wiseman, 2008).

[10]This approach requires the precise of knowledge of the (discrete) type distribution; see Levin and Pesendorfer (1995).

appropriate all the surplus. The instance of the Coase conjecture in the resource model requires not only frequent trades (i.e., continuous-time actions) but also extreme patience (i.e., no discounting); the standard requirement for the conjecture to hold is only frequent trades. An other difference between our model and Kahn is that the equilibrium timing of the substitute adoption takes place in finite time, and therefore we developed an approach that departs from the one in Kahn.

# 4 Concluding remarks

We believe it is a fruitful agenda to further explore elements that may shape strategic interactions in resource markets. For example, in this paper we adopted the traditional and somewhat stark view on the substitute arrival. Once the choke price is reached, the substitute enters the market with perfectly elastic supply. Recent research has developed a multi-sector description of the resource substitution process such that the transition is gradual as sectors move substitutes at different times (e.g., Chakravorty, Roumasset, and Tse 1997). Gerlagh and Liski (2011) have shown that adjustment costs, in the form of a time-to-build period for the substitute, can bring about considerable bargaining power to the buyer side of the resource market —to the extent that sellers have to increase supplies over time. This, again, is a resource-market specific addition to the Coase conjecture discussion.

Another interesting area to be explored further is the buyer power in renewable-resource markets, and here too it should be possible to learn from the well-explored Coase conjecture. For example, Bond and Samuelson (1984) consider, using the resource terms, "a renewable consumer stock". The connections to the renewable resource are yet to be explored.

# 5 Appendix : Commitment solution

We derive here the commitment solution for the resource monopsonist when he cannot discriminate between resource and substitute suppliers. Assume $c < \bar{p} = U'(\bar{q}) < U'(0)$, as in the text, and use the definition $W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$ for the long-run surplus flow.

The buyer's problem can be written as

$$
\begin{aligned}
V &= \max_{\{q_t,T\}} \int_0^T \{U(q_t) - p_t q_t\} e^{-\delta t} dt + \frac{1}{\delta} e^{-\delta T} W(\bar{q}) \\
& \quad s.t. \\
\frac{dQ_t}{dt} &= -q_t, Q_0 > 0, Q_T = 0. \\
\frac{dp_t}{dt} &= \delta(p_t - c), p_T = \bar{p} > 0
\end{aligned}
$$

This problem is well defined due to our assumption on the objective functions and the linearity of the state equations (Seierstad and Sydsaeter, 1988). Thus, the first-order conditions are also sufficient for the optimum. To obtain these conditions, write the current-value Hamiltonian as

$$\mathcal{H}_t = U(q_t) - p_t q_t - \lambda_t q_t + \eta_t \delta(p_t - c)$$

where $\lambda_t$ and $\eta_t$ are the co-states of $Q_t$ and $p_t$, respectively. The (interior) optimality conditions include

$$\frac{\partial \mathcal{H}_t}{\partial q_t} = U'(q_t) - p_t - \lambda_t = 0 \tag{18}$$

$$\frac{d\lambda_t}{dt} = \delta\lambda_t - \frac{\partial \mathcal{H}_t}{\partial Q_t} = \delta\lambda_t \tag{19}$$

$$\frac{d\eta_t}{dt} = \delta\eta_t - \frac{\partial \mathcal{H}_t}{\partial p_t} = q_t. \tag{20}$$

In addition, a boundary condition for choosing the optimal $T$ is needed. The Hamiltonian evaluated at $T$ measures the marginal value of increasing $T$, so that the present-value effect is $e^{-\delta T}\mathcal{H}_T$. On the other hand, the marginal value of postponing the arrival of the substitute surplus is $-e^{-\delta T} W(\bar{q})$, evaluated at $t = 0$. The optimal $T$, chosen at $t = 0$, equates the two marginal effects, that is

$$\frac{\partial V}{\partial T} = e^{-\delta T}\mathcal{H}_T - e^{-\delta T} W(\bar{q}) = 0. \tag{21}$$

Conditions (18)-(21) are enough for determining the solution; the boundary values of the states are fixed, so they require no transversality conditions. Combining (18) and (19) gives

$$c = U'(q_t) - \frac{1}{\delta} U''(q_t) \frac{dq_t}{dt} \tag{22}$$

consistent with the consumption equation (4) in the text. In addition, condition (20) implies

$$\eta_t = Q_0 - Q_t. \tag{23}$$

Using the definition of $\mathcal{H}_T$ and $p_T = \bar{p}$ together with (18), (19) and (23), allows us to write (21) as

$$\{U(q_T) - U'(q_T)q_T\} + \delta(\bar{p} - c)Q_0 - W(\bar{q}) = 0. \tag{24}$$

Rearranging leads to the stopping condition in the text. Note that (22) is a well-defined first-order differential equation with a boundary value given by $q_T$ satisfying (24).

# 6 Appendix B: The extensive form

Assume a finite number of periods $i = 1, ..., N < \infty$ for the interaction between the buyer and the resource market; after $N$ the interaction stops and the buyer consumes $\bar{q}$ units of the substitute indefinitely. Each period lasts $\Delta > 0$ units of time.

The buyer's strategy is a collection of consumption functions

$$C = (C_1(\cdot), ..., C_N(\cdot))$$

that defines consumption $q_i = C_i(h_i)$ at any period $i$ as a function of the history $h_i$ at $i$, where

$$h_i = ((q_1, p_1), , .., (q_{i-1}, p_{i-1})) \in \mathbb{R}_+^{2(i-1)}$$

for $i > 1$.

Similarly, market price is given by a collection of price functions

$$P = (P_1(\cdot), ..., P_N(\cdot))$$

that depend on $h_i$ and the buyer's choice $q_i$, i.e., $P_i(h_i, q_i)$. These price functions will be constructed from the structure of the economy such that the sellers will be indifferent between selling at stage $i$ and saving their stock for a later stage $j > i$.

A given profile $(C, P)$ generates consumption, price and stock paths, that is, $q_i = C_i(h_i)$, $p_i = P_i(h_i, C_i(h_i))$ and $Q_i = Q_0 - \sum_{j=1}^{i} \Delta q_j$, respectively, for all $i = 1, ..., N$. Note that $Q_0$ is the initial stock, $Q_{i-1}$ is the stock at the beginning of period $i$ and $Q_i = Q_{i-1} - q_i$ is the stock at the end of period $i$.

The payoff to the buyer is then a matter of accounting:

$$V_i(h_i, C, P) = \sum_{j=i}^{N} \Delta\{U(q_j) - p_j q_j\}e^{-\Delta\delta(j-i)} + \frac{e^{-\Delta\delta(N+1)}}{1 - e^{-\Delta\delta}} W(\bar{q}).$$

The long-run surplus is enjoyed after the end of the resource consumption game, but the buyer can choose to consume $\bar{q}$ units of the substitute at any stage before that (the substitute is available, at price $\bar{p}$, from the start of the game).

The subgame-perfect equilibrium of the game is defined as the profile $(C^e, P^e)$ that for any $h_i$, (i) $C^e$ maximizes $V_i(h_i, C, P^e)$ and (ii) $P^e$ satisfies the Hotelling condition

$$P_i(h_i, C^e) \geq e^{-\Delta\delta} P_{i+1}(h_{i+1}, C^e),$$

where the equality holds whenever $Q_i > 0$.

The equilibrium is found by backward induction. If the stock left at the final stage $N$ is equal or larger than the substitute supply, i.e., $Q_{N-1} \geq \bar{q}$, the buyer will demand slightly less than the remaining stock, and since sellers have no remaining trading opportunities, the resource price would collapse to zero. If, on the other hand, $Q_{N-1} < \bar{q}$, the buyer's optimal demand is either (i) consume $\bar{q}$ at price $\bar{p}$, or (ii) buy the remaining stock $Q_{N-1} - \varepsilon$ at zero price with $\varepsilon$ arbitrarily small. The former occurs if and only if $W(\bar{q}) \geqslant U(Q_{N-1} - \varepsilon)$. The best-responses and equilibrium prices are thus well defined functions of the stock at stage $N$. The same reasoning applies to stage $N-1$ except that the sellers continuation price is given by the remaining trading opportunity at stage $N$. Proceeding this way one can verify that the payoff-relevant history is fully summarized by the remaining stock, that is,

$$C_i(h_i) = C_i(Q_{i-1}) \text{ and } P_i(h_i, q_i) = P_i(Q_{i-1} - q_i) = P_i(Q_i). \tag{25}$$

Consider now the limiting case where $\Delta$ is small. We want to state a relationship between $N$ and $\Delta$ such that there is scarcity over the $N$ potential consumption periods.

**Remark 1** *For finite $N$ satisfying $N > Q_0/\Delta z$, where $W(\bar{q}) = U(z)$, the equilibrium interaction stops at $M < N$.*

To establish the lower bound for the length of the game that introduces resource scarcity, consider a constant consumption flow of $z < \bar{q}$ and $\underline{N}$ such that $W(\bar{q}) = U(z)$ and $Q_{N-1} = Q_0 - (\underline{N} - 1)\Delta z = \Delta\bar{q}$. From the arguments above, the flow $z$ would be consumed at zero price if $N \leq \underline{N}$. We can then define the lower bound for the number of strategic interactions needed for scarcity as $N > Q_0/\Delta z$. The buyer cannot consume $z$ throughout because there is not enough resource for doing so ($\Delta z N > Q_0$). The resource will thus be exhausted before $N$, and as a result the price that will prevail in the market before the interaction stops is $\bar{p}$. Now, letting $\Delta$ approaches zero while satisfying $N > Q_0/\Delta z$ preserves the property that the stock is the only payoff relevant variable.

In the text we construct a stationary equilibrium with these properties in the continuous-time limit. That such a stationary equilibrium emerges is consistent with Maskin and Tirole's (2001) requirement for Markov perfection in the sense that strategies should be identical in states where the continuation payoffs are identical. In a game of a few periods, strategies surely depend on calendar time since the continuation game is not the same at different periods, even if the stock were the same. Furthermore, the stationarity property should remain as we transit from the discrete-time to the continuous-time formulation as long as Remark 1 holds, that is, as long as the equilibrium strategic interaction endogenously stops before $N$. In such a case the precise value of $N$ is irrelevant for the payoffs, so it is not surprising that the equilibrium strategies we find do not depend on calendar time.[11]

# References


[1] Ausubel, L., and R. Deneckere. 1989. "Reputation and Bargaining in Durable-Goods Monopoly", *Econometrica* 57, 511-32.

[2] Bagnoli, M., S. W. Salant, and J. E. Swierzbinski. 1989. "Durable-Goods Monopoly with Discrete Demand", *Journal of Political Economy* 97, 1459-1478.

[3] Bergstrom, T. C. 1982. "On Capturing Oil Rents with a National Excise Tax", *American Economic Review* 71, 194-201.

[4] Bulow, J. I. 1982. "Durable-Goods Monopolists", *Journal of Political Economy* 90, 314–32.

[5] Bond, E. W., and L. Samuelson. 1984. Durable good monopolies with rational expectations and replacement sales, *Rand Journal of Economics* Vol. 15, No. 3.

[6] Chakravorty, U., J. Roumasset and K. P. Tse. 1997. "Endogenous Substitution of Energy Resources and Global Warming", *Journal of Political Economy* 105, 1201-1233.

[7] Coase, R. H. 1972. "Durability and Monopoly", *Journal of Law and Economics* 15, 143–49.

[8] Daubanes, J., and P. Lasserre. 2011. "Optimum Commodity Taxation with a Non-Renewable Resource", CIRANO working paper, 2011s-05.


[11] Gerlagh and Liski (2011) prove this formally for a game that shares the same stationarity property. They do it by induction around the equilibrium stopping period.

[9] Dasgupta, P. S., and G. M. Heal. 1979. "Economic Theory and Exhaustible Resources ", Cambridge: Cambridge Univ. Press.

[10] Gerlagh, R., and M. Liski. 2011. "Strategic Resource Dependence", *Journal of Economic Theory*, forthcoming.

[11] Gul, F., H. Sonnenschein, and R. Wilson (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture", *Journal of Economic Theory* 39, 155-190.

[12] Hotelling, H. 1931. "The Economics of Exhaustible Resources", *Journal of Political Economy* 39, 137–75.

[13] Hörner, J., and M. Kamien. 2004. " Coase and Hotelling: A Meeting of the Minds", *Journal of Political Economy* 112, 718-723.

[14] Kahn, C. M. 1986. "The Durable Goods Monopolist and Consistency with Increasing Costs", *Econometrica* 54, 275–94.

[15] Karp, L. 1984. "Optimality and Consistency in a Differential Game with Non-Renewable Resources", *Journal of Economic Dynamics and Control* 8, 73-97.

[16] Karp, L. 1996. "Depreciation Erodes the Coase Conjecture ", *European Economic Review* 40, 473-490.

[17] Karp, L., and D. Newbery. 1993. "Intertemporal Consistency Issues in Depletable Resources",˙In Handbook of Natural Resource and Energy Economics, vol. 3, edited by Allen V. Kneese and James L. Sweeney. Amsterdam: Elsevier Science.

[18] Levine, D. K., and W. Pesendorfer. 1995."When Are Agents Negligible ", *American Economic Review* 85, 1160-1170.

[19] Liski, M., J.-P. Montero. 2011. "Market Power in an Exhaustible Resource Market: The Case of Storable Pollution Permits", *The Economic Journal*, forthcoming.

[20] Maskin, E., and D. Newbery. 1990. "Disadvantageous Oil Tariffs and Dynamic Consistency ", *American Economic Review* 80, 143-156.

[21] Maskin, E., and J. Tirole. 2001. "Markov Perfect Equilibrium I: Observable Actions", *Journal of Economic Theory* 100, 191-219.

[22] McAfee, P., and T. Wiseman. 2008. "Capacity Choice Counters the Coase Conjecture", *Review of Economic Studies* 75, 317-332.

[23] Nordhaus, W., 1973. "The Allocation of Energy Reserves", Brookings Papers 3, 529-570.

[24] Seierstad, A., and K. Sydsaeter. 1987. Optimal Control Theory with Economic Applications, Amsterdam: North Holland.

[25] Sobel, J. 1991."Durable Goods Monopoly with Entry of New Consumers", *Econometrica* 59, 1455-1485.

[26] Stokey, N. L. 1981. "Rational Expectations and Durable Goods Pricing", *Bell Journal of Economics* 12, 112–28.

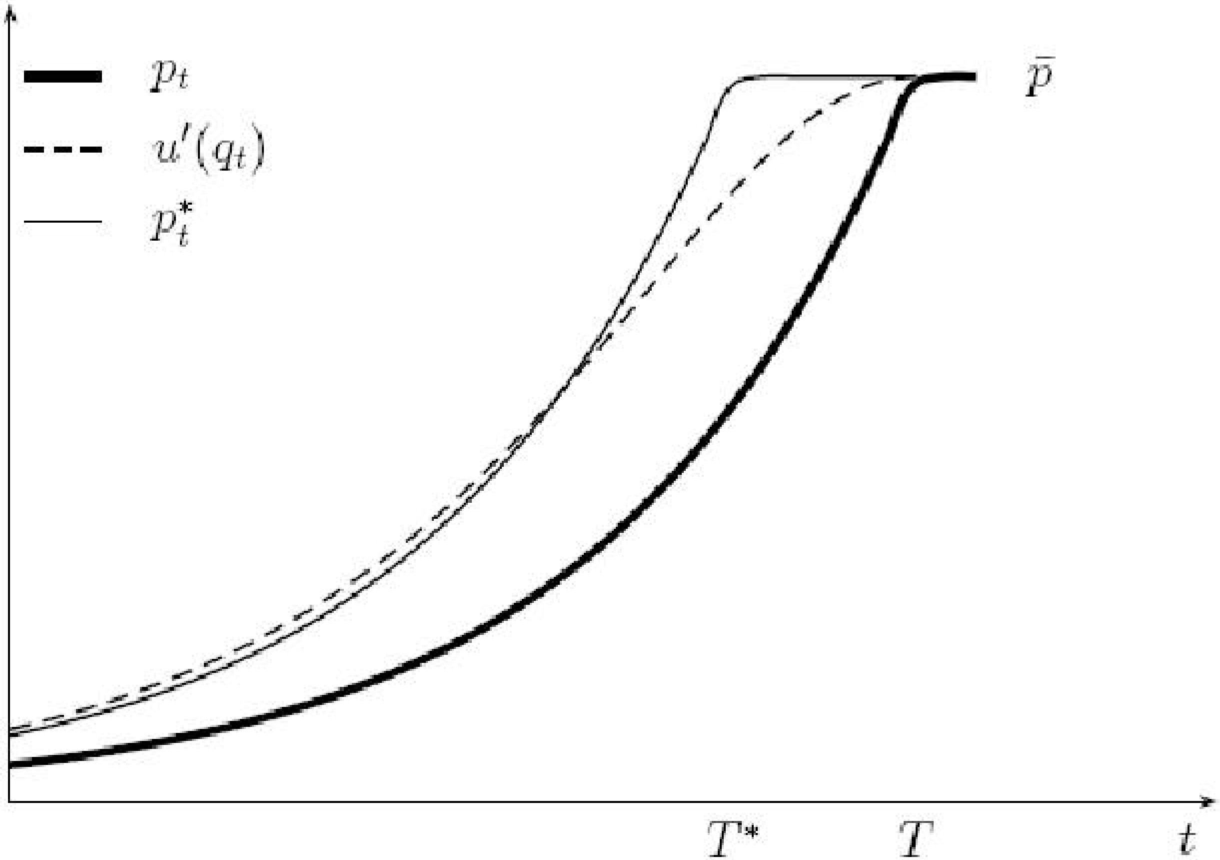


Figure 1: Equilibrium price, marginal utility, and the first-best price path