On Coase and Hotelling

Matti Liski and Juan-Pablo Montero*

September 17, 2009

Abstract

It has been long argued that the connection between the durable-goods monopoly and the exhaustible-resource monopsony provides a basis for understanding the design of policies targeted at influencing resource imports and consumption. We characterize how the surplus is divided in the resource model, and show that the connection to durable goods breaks down under the standard assumption that the final value of the resource is determined by the cost of the (inexhaustible) substitute supply; there is no natural counterpart of such an element in the durable-goods theory. As a result, the surplus is shared differently in the resource model: competitive agents capture a fraction of the surplus even in the absence of heterogeneity in private valuations. Furthermore, the Coase conjecture (full loss of monopsony power) requires not only lack of commitment but also extreme patience.

JEL Classification: D42; L12; Q30.

Keywords: durable goods, exhaustible resources, Coase conjecture

---

*Liski <liski@hse.fi> is at the Economics Department of the Helsinki School of Economics and Montero <jmontero@faceapuc.cl> is at the Economics Department of the Pontificia Universidad Catolica de Chile. Both authors are also Research Associates at the MIT Center for Energy and Environmental Policy Research. Liski acknowledges funding from Yrjö Jahnsson Foundation and Montero from Fondecyt and Instituto Milenio SCI. We thank Preston McAfee, Juuso Välimäki, Glen Weyl and seminar participants at Columbia, Ecole Polytecnique, MIT, Bocconi, and Econometric Society Meeting (Rio 2008) for helpful discussions.

1 Introduction

The theory on durable-goods monopoly, originated with the Coase's conjecture, is one of the best understood models of dynamic competition in economics.[1] It would be of great value if the implications of a structure so well explored could be exported to other fields of economics. An important field where the Coase's insight seems to apply is the theory of exhaustible resources, put forward by Hotelling (1931). In the durable-goods problem, the monopolist faces a ceiling on cumulative demand as the market saturates. In the resource problem, the strategic agent is a single buyer (e.g., a policy maker interested in maximizing consumer surplus) that faces a ceiling on cumulative supply which comes from the exhaustible nature of the supply. This potential connection between the two problems—the durable-goods monopoly and the exhaustible-resource monopsony— is important, because it can provide a basis for analyzing the resource-dependence issue as a dynamic bargaining problem between the importing party and the suppliers in the Coasian spirit.

Maskin and Newbery (1990) and Karp and Newbery (1993) already noticed that strategic agents in both problems face similar time-consistency restrictions. More recently, Hörner and Kamien (2004) showed that the two problems are formally equivalent in the case where the strategic agents can commit. We know more generally, that the Coase conjecture in the durable-goods problem can arise only if consumer valuations decline with the stock of the good in the market. If lower valuation consumers are expected to enter the market at some future date, potential buyers will postpone purchases and wait for lower prices. According to the conjecture, when transactions are highly frequent, the monopolist will sell at a price equal to the valuation of the lowest-valuation consumer. In the resource monopsony, the stock is the amount of the resource already extracted, and the private value changing with the stock is the sellers' extraction cost. The conjecture would then read as follows: if low-cost sellers can wait for high-cost sellers to enter the market, they will do so and thereby force the buyer to pay a price equal to the cost of the highest-cost supplier. Accordingly, the buyer's monopsony power vanishes

[1]The conjecture was presented in Coase (1972). In the literature that follows, the conjecture is understood as the entire loss of monopoly power when consumers are patient enough. Early formalizations are Stokey (1981), and Bulow (1982). The monopolist may escape the conjecture, at least partially, if: marginal production costs are convex (Kahn, 1986); reputational strategies can be used (Ausubel and Deneckere, 1989); a price-quantity scheme can be used to discriminate among discrete buyers (Bagnoli et al., 1989); the good depreciates (Karp, 1996); there is entry of new consumers (Sobel, 1991); or there are capacity costs (McAfee and Wiseman, 2008).

"in the twinkling of an eye", as expressed by Coase for the durable-goods monopoly.

Our result is that the above analogy does not hold. We find that the resource version of the Coase conjecture is fundamentally different provided the final value of the resource (i.e., choke price) is determined by the cost of an inexhaustible substitute supply. In resource economics, this is the prevailing, if not the only, interpretation of the choke price (see, e.g., Dasgupta and Heal, 1974 and 1979). Substitutes that put a cap on resource prices are often called backstop technologies, after Nordhaus' (1973) work on the effect of future backstops on resource prices. A switch to the backstop or alternative supply implies that the resource stock will be consumed during a finite period of time. According to the durable-goods analogy, resource suppliers would take a larger share of the resource surplus the faster the rise in costs is. Conversely, when extraction costs are constant (i.e., independent of the evolution of the stock), the buyer should achieve his first-best outcome and appropriate the whole surplus.

We show, however, that the surplus-sharing in the resource market follows a different logic than that in the durable-goods market if the buyer eventually switches to the substitute *at some future date*. We find that the resource sellers will always capture some surplus due to their ability to wait for the buyer's outside option (inexhaustible substitute), even in the absence of extraction costs. This is in sharp contrast with the durable-goods monopoly theory where consumers' heterogeneity is the driving force behind the Coase conjecture. It turns out that none of the existing durable-good models, or their reinterpretations, can explain the surplus-sharing arising in this, arguably, basic version of the resource model.[2]

The determinants of the Coase conjecture in the resource problem (i.e., suppliers capture the full surplus from resource consumption) are different from those in the durable-

[2]One way to illustrate the difference between the two theories is by looking at the "gap" case. There is a gap in the durable-good model when the valuation of the lowest-valuation consumer is strictly above the monopolist's marginal cost, and in the exhaustible-resource model when the unit cost of the highest-cost seller is below the monopsonist's choke price. We know (see, e.g., Gul et al., 1986) that the durable-good monopoly has an equilibrium where the terminal price equals the lowest valuation (when there are very few potential buyers left, it pays the monopolist to serve them all at a price equal to the lowest valuation which is strictly above marginal cost. This result pins down the price in the last stage and ensures that the game is of finite length). According to this durable-good logic, the terminal price in the exhaustible-resource monopsony would be the highest unit cost, which is lower than the choke price. But right after consuming the last unit of the resource the monopsonist is buying from the alternative supply at the choke price, so that supplying the last unit of the resource for something less than the choke price cannot be part of an equilibrium.

goods theory. The conjecture for the durable-goods monopoly says that as transactions become highly frequent the (constant-cost) monopolist loses all its commitment power and ends up serving the market rather quickly, almost instantly, in real time. Therefore, discounting plays no role (it does, as we will see shortly, when the conjecture is partially prevented as in Kahn (1986)). In the resource model, discounting plays a crucial role, since the arrival of the substitute comes after the physical depletion of the resource, and it may take long in real time for this to happen even with very frequent transactions. But a long depletion period does not necessarily prevent the conjecture from happening (as it would in a durable-goods market). Indeed, the conjecture is approached as discounting disappears, and agents become extremely patient. The conjecture arises from the presence of the substitute alone and not from the conditions typically associated to the durable-goods problem (frequent transactions, constant costs and declining valuations). This result can still be named after Coase since it arises from the sellers' ability to wait for the strategic agent's outside option to materialize; hence, the mechanism follows Coase's reasoning.

To further contrast the our Coasian result with the one for durable goods, it helps to consider Kahn (1986) who shows that the durable-goods monopolist can recoup some commitment when the production technology allows him to smooth production. The resource model is a variant of Kahn's model with production smoothing replaced by consumption smoothing (i.e., concave utility function). As discounting vanishes in Kahn (1986), the monopolist can almost perfectly price discriminate among consumers (pp. 287-288). The exact opposite, however, occurs in the resource model: competitive agents (sellers) appropriate all the surplus.

With positive discounting (and no commitment), the way the surplus is divided between the resource buyer and the sellers depends on how important switching to the substitute is for the buyer. The less profitable the switch is, the better the chances the buyer has to exercise his monopsony power and depress prices. When switching to the substitute provides no benefit to the buyer, the world ends at the exhaustion of the resource and the buyer can commit to his first-best outcome as long as there is no cost heterogeneity among sellers. In this limiting case (i.e., when there is no benefit from the substitute), the resource model coincides with the durable-goods model, which is the case considered by Hörner and Kamien (2004). On the other hand, when the substitute is important, the buyer's ability to credibly delay consumption and depress prices diminishes because he does not want to indefinitely postpone the arrival of the valuable substitute. In this case, sellers capture a larger fraction of the resource surplus.

The market for oil has motivated much of the literature on optimal tariffs on exhaustible resources (e.g., Maskin and Newbery, 1990; Karp and Newbery, 1993), but "market power" on the buyer side can also arise from policies targeted at reducing fossil-fuel use due to externalities such as those related to climate change. The durable-goods analogy would imply a large potential for extracting the sellers' resource rent when the oil production cost is largely independent of the stock level. This situation best describes the conventional, cheap oil stock which is mainly held by the OPEC group. One may argue that the stock of conventional oil is the exhaustible-resource in the oil market, and the nonconventional oils are rather the substitute commodities. In such situation, our results suggest a conclusion that is quite different from that of the durable-goods theory. According to our results, the cheap oil producers will receive a price comparable to the cost of supplying the substitute, not their own cost. The buyer's side effort to coordinate demand reduction through tariffs or other policies can depress the price, but they do so only by delaying the arrival of the substitute.

We organize the rest of the paper as follows. We build on Kahn's (1986) framework as described in Section 2. This model is a natural choice as it embeds common interpretations for both the resource and the durable-goods problems. We start by describing the first-best outcome for the strategic agent (the commitment solution) and show that the disconnection between the two models is already visible in this solution. This approach also helps to connect explicitly to Hörner and Kamien (2004) who consider the commitment solution under the assumption that the world ends when the resource is exhausted. In Section 3, we derive the equilibrium path without commitment. To sharply isolate the source of differences between the models, we start with a durable-goods setting where the monopolist faces no commitment problem. We then demonstrate that in the corresponding resource model the commitment problem remains (due to the different nature of the strategic agent's outside option). We also explain how the Coase conjecture can arise in the resource model. We end the section showing that the difference between the models holds quite generally. We conclude in Section 4 with a discussion of how the durable-goods model needs be modified to restore its connection, if at all, to the resource model.

2 Commitment solutions

As in Kahn (1986), the durable-goods monopolist is a single producer of a perfectly durable good. For exposition, we assume a perfect re-sale or rental market.[3] The monopolist sells the good by giving away the property right, but subsequent users may also rent it. The flow valuation of the service provided by the good is a function of the total cumulative stock of the good available at time t. We denote the stock by S_t and the flow valuation by $P(S_t)$ which is a monotone non-increasing function defined on $[0, \bar{S}]$. There is a continuum of potential buyers. If the path $(S_\tau)_{\tau \geq t}$ is known, the market clearing price at any time t is given by

$$p_t = \int_t^\infty P(S_\tau)e^{-\delta(\tau-t)}d\tau \tag{1}$$

where δ is the discount rate. The price of the durable good is thus the discounted value of the rental service it provides.

The monopolist has a strictly convex cost of production $\gamma(q_t)$ that depends on the rate of production $q_t = dS_t/dt$. If the monopoly can commit to a path $(S_\tau)_{\tau \geq t}$ at $t = 0$, it will choose this path by solving

$$\max_{q_t} \int_0^\infty \{p_t q_t - \gamma(q_t)\}e^{-\delta t}dt$$

subject to (1), $q_t = dS_t/dt$, and $S_0 \geq 0$. Using (1), this problem can be rewritten as

$$\max_{q_t} \int_0^\infty \{P(S_t)(S_t - S_0) - \gamma(q_t)\}e^{-\delta t}dt.$$

The first-order condition implies that positive sales satisfy

$$P(S_t) + P'(S_t)(S_t - S_0) = \delta\gamma'(q_t) - \gamma''(q_t)\frac{dq_t}{dt}. \tag{2}$$

The left-hand side of (2) is the marginal revenue from renting an additional unit, and the right-hand side is the marginal cost of producing that unit today rather than tomorrow.

For the Hotelling monopsony, we assume that there is a single importer (buyer) of an exhaustible resource. The buyer's utility depends on the rate of consumption q_t. We denote his utility by $U(q_t)$ which is strictly concave. Each supplier has one unit of the resource and a given cost of extracting and selling that unit. We assume a continuum of

[3] The existence of a rental market is inconsequential when there is a continuum of agents. We can assume that each consumer buys either one or zero unit of the good, and disappears after purchase. Alternatively, buyers can be intermediaries who use the goods to serve the resale or rental market.

suppliers indexed by $S \in [0, \bar{S}]$ and that the unit cost depends on this index. The unit cost is given by a nondecreasing function $c(S_t)$. In continuous time, $c(S_t)q_t$ is the cost of extracting at rate q_t when the stock already extracted or consumed is S_t (see, e.g., Karp and Newbery 1993).

For a given path $(S_\tau)_{\tau \geq t}$, market clearing requires that at all times where sales take place, the sales price satisfies the following arbitrage condition

$$\frac{dp_t}{dt} = \delta(p_t - c(S_t)),$$

which is the Hotelling rule. After some manipulation, it can be rewritten as

$$p_t = e^{\delta t}(K - \int_0^t \delta e^{-\delta\tau} c(S_\tau) d\tau) \tag{3}$$

where K is a constant of integration that corresponds to some initial price p_0. Assuming that equilibrium p_t is bounded by some finite value, we can let $t \to \infty$ and evaluate K as

$$K = \int_0^\infty \delta c(S_t) e^{-\delta t} dt \tag{4}$$

which leads to

$$p_t = \int_t^\infty \delta c(S_\tau) e^{-\delta(\tau - t)} d\tau. \tag{5}$$

Expression (5) indicates that the price at any time t is given by the present value of the future unit cost of extraction. Note already that (5) looks remarkably similar to (1) but for the "symbols".

If the resource monopsony can commit to a path $(S_\tau)_{\tau \geq t}$ at $t = 0$, it will choose this path by solving

$$\max_{q_t} \int_0^\infty \{U(q_t) - p_t q_t\} e^{-\delta t} dt$$

subject to (3), $q_t = dS_t/dt$, $S_t \leq \bar{S}$, and $S_0 = 0$. Using (5), we can rewrite the objective function as

$$\int_0^\infty \{U(q_t) - \delta c(S_t) S_t\} e^{-\delta t} dt.$$

Over the interval of time where sales are positive, the optimal consumption must satisfy the first-order condition[4]

$$c(S_t) + c'(S_t) S_t = U'(q_t) - \frac{1}{\delta} U''(q_t) \frac{dq_t}{dt}. \tag{6}$$

[4]In terms of the remaining stock, denoted by $Q_t = \bar{S} - S_t$, eq. (6) becomes (see, e.g., Karp and Newbery, 1993, p. 894)

$$c(Q_t) - c'(Q_t)(Q_0 - Q_t) = U'(q_t) - \frac{1}{\delta} U''(q_t) \frac{dq_t}{dt}$$

where $Q_0 = \bar{S}$, $dQ_t/dt = -q_t$, and $c'(Q_t) \leq 0$.

The left-hand side of (6) is the marginal cost from buying an extra unit and the right-hand side is the marginal benefit of consuming that unit today rather than tomorrow.

In view of the conditions (2) and (6) and price functions (1) and (5), the equivalence noted by Hörner and Kamien (2004) is apparent: renaming $P(S)$ as $-\delta c(S)$ and $\gamma(q)$ as $-U(q_t)$, shows that the commitment solutions differ only in the interpretations placed on symbols. Note in particular that when the cost function $\gamma(q)$ is strictly convex, the Coase monopoly has preferences for production smoothing. This corresponds to preferences for consumption smoothing in the Hotelling monopsony, arising when the utility function is strictly concave.

The Coasian commitment problem is known to arise because of declining consumer valuations, that is, because $P'(S) < 0$. Potential consumers can benefit from waiting only if lower valuation consumers are anticipated to enter the market in the future. This source of time inconsistency can be formally seen from equation (2). The initial stock S_0 only plays a role there if $P'(S) \neq 0$. If the monopoly could reconsider at some future date $t > 0$ his original plan $(S_\tau)_{\tau \geq t}$ announced at $t = 0$, the initial condition would change from S_0 to $S_{t>0}$, which would result in a new (more competitive) solution.

The above reasoning suggests that the problem of commitment arises in the resource model only when extraction costs $c(S_t)$ are strictly increasing. According to the conjectured analogy, low cost sellers can benefit from delaying sales only when high cost sellers are anticipated to enter the market in the future. In other words, if $c'(S_t) = 0$, the consumption dynamics solved at any future date $t > 0$ are identical to those solved at $t = 0$. In this case the buyer has no incentives to deviate from the consumption path that he announced at $t = 0$.

The above analysis, however, ignores the fundamental difference between the two theories that is introduced when it is assumed that the resource buyer switches to an alternative supply at some future date. Suppose the buyer's benefit from switching to the alternative supply is $W > 0$ per period. In our setting, this benefit can be captured as

$$W = W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q},$$

where $\bar{p} = U'(\bar{q})$ is the unit price of the alternative supply —also known as the choke price— and $\bar{q}$ is the amount of alternative supply measured in exhaustible-resource equivalents. We argue that as long as $W(\bar{q}) > 0$, there is a source of time-inconsistency unrelated to extraction costs, no matter how small $W(\bar{q})$ is. And if this is the case, the price path is no longer governed by equation (5).[5]

Before characterizing more formally the equilibrium path in the resource model, let us note that the difference between the models is already visible in the commitment solutions. For example, when $c(S_t) = c$, the price path for the commitment solution is given by

$$p_t = e^{\delta t}[p_0 - c] + c$$

for all $p_t < \bar{p}$. The initial price p_0 is a choice variable for the monopsonist in that its level can be chosen by the shape of the consumption path. In particular, the buyer prefers to commit not to use the substitute for a long time if the backstop utility is negligible, i.e., if $W(\bar{q})$ is close to zero. In this case, the buyer will commit to a path $(S_\tau)_{\tau \geq 0}$ that postpones the consumption of an ε-amount —an arbitrarily small amount— of the resource for far enough into the future.[6] This destroys the sellers' option of waiting: p_0 collapses to cost c as the sellers race for early sales. Clearly, if such commitment is not sequentially rational, sellers cannot be left with no rents.[7]

3 The difference between the models

In the resource model it is natural to have the stock be gradually consumed over time and the switch to the substitute supply taken place at some future date. These properties are ensured by the strictly concave utility function and a finite choke price above costs. For durable goods, the corresponding specification exhibits strictly convex production cost $\gamma(q)$, and $P(\bar{S}) > \gamma(0) = 0$. From Kahn (1986), we know that the durable-goods seller's commitment problem arises from the changing consumer valuation, not from convex costs. One may thus conjecture that the monopoly can achieve its first-best in equilibrium when the consumer valuation is constant but costs are still convex. For the resource model, the analogous specification favoring the strategic buyer is one in which extraction costs are constant while the utility is strictly concave.

[5]Recall that in constructing (5) we never imposed $\bar{p}$ as the terminal price which, however, must hold if the substitute enters at price $\bar{p}$.

[6]In the durable-good equivalent, that is, with $P'(S) = 0$, the monopolist does not need leave an ε-fraction of consumers for late delivery in order to implement its first-best.

[7]Throughout this paper, we will make arguments that exploit the assumption that there is a continuum of small sellers. This is justified by the arguments given in Levin and Pesendorfer (1995). Also, the arguments where we allocate ε-amounts of agents across periods do not imply existence problems, any more than similar arguments in connection with Bertrand competition, for example.

To isolate the commitment problem coming from the substitute price, it helps to state the difference between the two models using constant valuations for the durable goods problem and constant costs for the exhaustible resource problem. We will show that the bargaining powers emerge in opposite ways in the two models. Moving to increasing stock-dependent costs does not eliminate the sellers' bargaining power coming from the substitute price; on the contrary, it is expected to reinforce it.

3.1 The durable-good benchmark

Assume thus for the durable-goods model the following variant of the Kahn's (1986) framework: the consumer valuation is constant, $P(S) = v$ for all $S \in [0, \bar{S}]$, and the production cost $\gamma(q)$ is assumed to be strictly convex. Note that there is a continuum of consumers. We consider a subgame-perfect equilibrium, and we want to assume discrete time periods to make the extensive form of the game clear. Discrete periods extend to infinity, $t = 0, 1, 2, ...$ At each period t, the monopolist chooses q_t, and after this, the competitive market clears the price p_t.

We look for a sales strategy $q_t = q_t(h_t)$ that depends on the history h_t at each t where

$$h_t = ((q_0, p_0), (q_1, p_1), ..., (q_{t-1}, p_{t-1})) \in R_+^{2t}.$$

The pricing strategy for the market is a function of the history and the seller's current choice, $p_t = p_t(h_t, q_t)$. Finding the equilibrium for this specification is a simple undertaking. We verify that the monopolist's first best outcome is a subgame-perfect equilibrium. The result holds for any period length, implying that the commitment built into the period length is not important.

In discrete time, the commitment solution is a sequence $\{S_t\}_{t=0}^N$ such that (i) $S_N = \bar{S}$, (ii) the marginal profit is the same from each period in present value, and that (iii) it is not optimal to extend the sales period from N. The last two requirements imply, respectively

$$\begin{aligned} v - \gamma'(q_t) &= \beta(v - \gamma'(q_{t+1})) \text{ for all } 0 \leq t < N, \\ v - \gamma'(q_N) &\geq \beta(v - \gamma'(0)), \end{aligned}$$

where $\beta = e^{-\delta\Delta}$ is the continuous-time discount factor over the period Δ (we can set $\Delta = 1$ here). The monopolist maximizes the overall social surplus. When the consumer valuation is constant, the seller has socially-optimal incentives for production smoothing. Note that in this discrete-time setting the market must be served in finite time, $N < \infty$, because condition (iii) must eventually hold as q_t strictly decreases over time.

Denote now the socially optimal production rule by $q_t^* = q^*(S_t)$. Note that when the equilibrium horizon is finite, it is sufficient to let sales depend exclusively on the current stock; hence, we can let $h_t = S_t$ be the payoff-relevant history (see, e.g., Kahn, 1986). Consider then the strategy $q_t(h_t) = q^*(S_t)$ for the seller, and $p(S_t, \cdot) = v$ for the market.[8] Clearly, the seller cannot have profitable one-shot deviations from $q^*(S_t)$. But neither has the buyer side of the market from $p(S_t, \cdot)$; no surplus is available in any conceivable continuation game.

For intuition, note that rental value v for the good remains even when the monopolist leaves the market. The market cannot resist buying the last units at that value, which gives the bargaining power to the seller. If there is no rental market and individuals leave the market at purchase, the conclusion remains the same. The seller can achieve the first-best by leaving an ε-mass of consumers unserved in the last period N. The price will jump to v, because no surplus is expected in continuation games as it is sequentially rational for the seller to follow the same strategy of leaving some remaining consumers unserved in all subsequent periods.[9] The welfare loss can be made arbitrarily small. And if instead of setting quantities the monopoly seller is setting prices, the first-best remains an equilibrium both with and without the rental market: the seller prices at v for all t and buyers consume along the efficient path.

3.2 The resource substitute and the Coase conjecture

Let us now turn to the equivalent exhaustible-resource monopsony with $c(S_t) = c$ for all $S \in [0, \bar{S}]$, strictly concave utility function $U(q)$, and a finite choke value $\bar{p} = U'(\bar{q}) > c$. The socially optimal allocation is a sequence $\{S_t\}_{t=0}^N$ that exhausts the stock at N (i.e., $S_N = \bar{S}$), equalizes the present value of marginal utilities net of costs, and keeps the latter above the choke value:

$$\begin{aligned} U'(q_t) - c &= \beta(U'(q_{t+1}) - c) \text{ for all } 0 \leq t < N, \\ U'(q_N) - c &\geq \beta(U'(\bar{q}) - c). \end{aligned}$$

This plan is also the monopsonist's first-best consumption plan if he can commit to it. When the substitute utility $W(\bar{q})$ is small, the buyer would like to commit to switch to the substitute far in the future. We have already explained how commitment can

[8] Note that technically speaking these strategies do not depend on t because they are stationary.

[9] If the seller brings the ε-amount to the market the remaining ε-consumers will bid the price down to zero.

transfer the full surplus to the buyer: an ε-amount is left for later consumption enough for destroying the "boundary value" in the sellers' price path. The buyer purchases at cost and consumes according to the first-best plan.

However, the buyer's commitment plan is not sequentially rational if $W(\bar{q}) > 0$ (i.e., $\bar{q} > 0$), not matter how small $\bar{q}$ and $W(\bar{q})$ are. For ease of presentation, from now on we will focus on the stock still left in the ground,

$$Q_t = \bar{S} - S_t.$$

Proposition 1 *Consider a given remaining stock Q_t, constant cost $c < \bar{p} = U'(\bar{q}) < U'(0)$, and period length $\Delta \to 0$. Then, (i) the resource surplus is split between the buyer and sellers as long as $W(\bar{q}) > 0$; (ii) the Coase conjecture arises as $\delta \to 0$ (and $W(\bar{q}) > 0$); and (iii) the buyer receives the full surplus as $W(\bar{q}) \to 0$.*

The Coase conjecture means that the buyer pays fully competitive prices. Without loss of generality we set $c = 0$ here and in the Appendix where we present the formal proof for the continuous-time consumption rule. (The results do not depend on whether the buyer sets quantities or prices in each period; we assume quantity setting in this proof.) We start by explaining the first result where the buyer and the sellers share the resource surplus under the more general conditions: positive discounting ($\delta > 0$) and substitute utility ($W(\bar{q}) > 0$). Results (ii) and (iii) follow as limiting cases.

Let us first explain what defines the respective bargaining powers of the buyer and sellers in a discrete time setting with $\Delta = 1$. Suppose that stock Q_0 is depleted in N periods. The first N prices are then

$$p_0 = \beta p_1 = ... = \beta^{N-1} p_{N-1} = \beta^N \bar{p} \tag{7}$$

In equilibrium, sellers must be indifferent between sales periods as long as there is some stock left.[10] If the buyer decides to delay the exhaustion of the stock, this can be achieved by reducing its demand by $\bar{q}$ units during the N first stages and saving these $\bar{q}$ units to stage $N+1$. This one period delay in the arrival of the substitute implies the new prices

$$p_0 = \beta p_1 = ... = \beta^N p_N = \beta^{N+1} \bar{p}. \tag{8}$$

[10]The last price p_{N-1} at stage $N-1$ equals the next period discounted choke price $\beta\bar{p}$ since the buyer demands some arbitrarily small ε less than the remaining stock to allocate some sellers to sell jointly with the substitute and, thereby, achieve the lowest possible price at stage $N-1$.

The consumption-cost difference between scenarios (7) and (8) is

$$\beta^N \bar{p}(Q_0 + \bar{q}) - \beta^{N+1}\bar{p}Q_0 = \beta^N(1-\beta)\bar{p}Q_0 + \beta^N\bar{p}\bar{q} \tag{9}$$

In both scenarios, consumptions and prices are the same after stage $N+1$, because $\bar{q}$ is consumed with price $\bar{p}$ in each period. We can thus focus on the difference in the first $N+1$ periods. In (7), the buyer consumes $\bar{q}$ in period $N+1$, but in (8) this consumption comes from the stock, which explains the last term in (9). The interpretation of (9) is then that by reducing consumption by $\bar{q}$ units over N periods, the buyer receives a price discount on the full stock, $\beta^N(1-\beta)\bar{p}Q_0$, plus the savings from postponing the purchase of the substitute in one period, $\beta^N\bar{p}\bar{q}$.

In equilibrium, the buyer chooses consumption and, thus, how much to delay the arrival of the substitute on a period-by-period basis; not over the entire consumption plan as described above. To get an idea of the costs and benefits of today's consumption choice, consider the first period equilibrium consumption q_0, and a one-shot deviation from the equilibrium path such that the full consumption q_0 is saved for later consumption. If the period length is $\Delta > 0$, the implied overall saving is Δq_0. Since we are considering a deviation from the equilibrium, the consumption sequence of the resource is delayed by exactly one period with no further effect on equilibrium choices. This gives the price gain per unit of consumption saved as

$$\beta^N(1-\beta)\bar{p}\frac{Q_0}{\Delta q_0} + \beta^N\bar{p}.$$

In the continuous-time limit, $\Delta \to 0$, this expression becomes

$$e^{-\delta T}\delta\bar{p}\frac{Q_0}{q_0} + e^{-\delta T}\bar{p} = p_0\delta\frac{Q_0}{q_0} + p_0,$$

where T is the equilibrium time to exhaustion, and p_0 is the current price that equals the discounted choke price. In continuous time, even small changes in current consumption will alter the overall depletion time, allowing us to express the cost of giving up current consumption as the current marginal utility loss. Therefore, at any time t before exhaustion, the continuous-time equilibrium condition that balances the buyer's costs and benefits of delaying consumption is

$$U'(q_t) = \delta p_t\frac{Q_t}{q_t} + p_t. \tag{10}$$

In Appendix we derive this equilibrium consumption rule formally. Note that the first term in right hand side of (10) includes the interest earning on total present-value purchases along the equilibrium path from time t onwards. That sum is divided by q to transform it into marginal units, relevant for the current consumption choice.

In view of the above, it is clear that the buyer's bargaining power arises from the ability to destroy overall surplus through delayed purchases. This incentive to delay consumption drives the wedge between the price and marginal utility shown in (10). Using the price arbitrage $dp_t/dt = \delta p_t$ together with the boundary conditions $p_T = \bar{p}$ and $Q_T = 0$, the stock depletion equation $dQ_t/dt = -q_t$, and condition (10), the equilibrium path is fully determined. Figure 1 depicts how the price and the marginal utility develop over time.[11] The Figure also shows the first-best price path p_t^* and the associated exhaustion time T^*.

Let us now discuss the limiting cases. Sellers can expect a surplus share due to their ability to wait for the substitute price, and when δ decreases the price path shifts up together with the marginal utility path. When $\delta \to 0$, patience is extreme and the Coase conjecture arises. Competitive sellers capture the full resource surplus; there is no reason to accept any price lower than the buyer's outside option.

As $\bar{q} \to 0$ (and $W(\bar{q}) \to 0$), the equilibrium converges to the outcome suggested by the durable-goods theory (Hörner and Kamien, 2004). To see this, it is useful to note that the equilibrium marginal utility path is S-shaped (see Figure 1). The S-shape is a result of different forces that change over time. As shown in the figure, during the early stage of the equilibrium, when the remaining stock is relatively large, the equilibrium consumption path is only slightly distorted from the first-best consumption path, i.e., the path where marginal utility equals p_t^*. During this early stage the buyer can credibly commit to stay close to the first-best because it is too costly for him to distort consumption relative to the gain in terms of price reduction. Sellers are still willing to supply these (almost) first-best quantities at lower than first-best prices because they correctly anticipate that future prices will inevitably fall below first-best levels as the remaining stock shrinks. In the later stage of the equilibrium path, when the remaining stock is much smaller and exhaustion is closer, it becomes less costly for the buyer to distort consumption. It is essentially in this later stage where the buyer delays consumption by balancing the gains from lower prices and the losses from postponing the switch to the alternative supply.

[11] The figure was plotted using $U(q) = log(q)$, which leads to an explicit solution for the (remaining) stock

$$Q_t = e^{\delta t}(Q_0 - \frac{1}{2\delta \overline{p}} e^{\delta T}) + \frac{1}{2\delta \overline{p}} e^{\delta (T-t)},$$

where T is found from the boundary condition $Q_T = 0$. The descriptive features of the equilibrium follow from this solution.

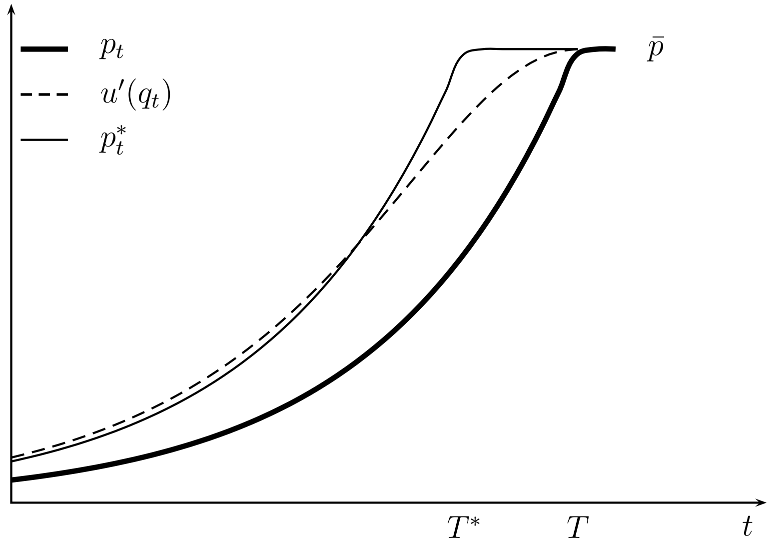

Figure 1: Equilibrium price, marginal utility, and the first-best price path

And the smaller the $W(\bar{q})$ is, the later the buyer can credibly start delaying consumption. As $W(\bar{q}) \to 0$, this delay becomes extreme, the stock is never exactly exhausted, and the price collapses to zero.[12]

It is natural ask how things change when extraction costs depend on the remaining stock, i.e., when unit cost increases with depletion, $c'(Q_t) < 0$. Using the boundary for the price path and the Hotelling rule, we can express the equilibrium price as

$$p_t = e^{-\delta(T-t)}\bar{p} + \int_t^T \delta c(Q_\tau) e^{-\delta(\tau - t)} d\tau.$$

The resource cannot be sold for anything less than $\bar{p}$ and, therefore, the equilibrium price converges to this level independently of whether the stock is economically (last units not extracted, $\bar{p} < c(0)$) or physically depleted (all units extracted, $\bar{p} > c(0)$). The equilibrium delay in buyer's consumption lowers the price path by postponing the arrival of the substitute much the same way as explained above —the substitute price appears independently of the cost structure in the price equation. This effect is a source of additional surplus going to the buyer; obviously, as long as the buyer is switching to the substitute at some point in the future (i.e., $W(\bar{q}) > 0$). For these reasons, the Proposition applies under more general cost structures.

[12]Note that the equilibrium solution when $W(\bar{q}) \to 0$ is no different than the commitment solution where the buyer leaves an ε-amount aside. It is sequentially rational for the buyer not to come to the market for this amount; if he does, the ε-suppliers will bid up the clearing price to virtually infinity since there is no alternative supply putting a cap on it.

4 Concluding remarks

We conclude by discussing some features of resource markets that may help the resource buyer to escape the conjecture, and how one might restore the equivalence between the resource and durable-good models. For the latter, one might ask what is the analogy of the resource substitute in the durable-good model? Recall that in the resource model, the substitute provides an outside valuation for the market (the choke price) with which the good can be ultimately sold. We have shown that in a subgame-perfect equilibrium the substitute changes the economic logic of the resource model in a fundamental way. It is therefore important to understand if a similar mechanism can be imported to the durable-good model.

To import the idea of the substitute to the durable-goods, one would need to assume two types of goods, durable and non-durable, such that the monopolist first serves the pool of customers buying the highly-valued durable good, and then switches to serve the (competitive) non-durable segment of the market for some flow profit of $W > 0$. Knowing the existence of the non-durable segment, which can always procure their non-durables at some competitive price, say $\underline{p}$, the monopolist would not be able to credibly commit to never attend the non-durable segment at price $\underline{p}$, which would ultimately prevent him from pricing its last durable units above $\underline{p}$. Thus, these "outside consumers" would in principle improve the bargaining position of the durable-good buyers, forcing the monopolist to leave a fraction of the durable-good surplus with the consumers (for example, when they have a constant valuation for the durable). It is immediately clear that this "backstop" interpretation is not at all that natural in the durable-goods case, while the substitute is an essential part of the resource model. Hence, there are good substance-related and economic reasons to argue that the two theories are distinct.

One may ask if any of the strategies that alleviate the Coase conjecture in the durable-goods model can significantly alter the implications of the resource model. It seems not. Ausubel and Deneckere (1989) result would require the buyer to never adopt the substitute. Strategies aimed at slowing down production, either through capacity constraints or convex costs (McAfee and Wiseman, 2008; Kahn, 1986), are already part of the resource model. The introduction of discrete agents (Bagnoli et al., 1989), the entry of new resource suppliers (Sobel, 1991), or depreciation (Karp, 1996) do not seem to eliminate either the role of the substitute in surplus sharing in the resource framework.

There is nevertheless a different way in which the buyer might be able to escape the conjecture, or more precisely, retain a larger share of the overall surplus. To maintain a close connection to the durable-good framework, in this paper we adopted the traditional and somewhat stark view on the backstop arrival. Once the choke price is reached, the substitute enters the market with perfectly elastic supply. Recent research has developed a multi-sector description of the resource substitution process such that the transition is gradual as sectors move substitutes at different times (e.g., Chakravorty, Roumasset, and Tse 1997). Gerlagh and Liski (2008) have shown that adjustment costs, in the form of a time-to-build period for the substitute, can bring about considerable bargaining power to the buyer side of the resource market —to the extent that sellers have to increase supplies over time. This, again, is a resource-market specific addition to the Coase conjecture discussion. We believe it is a fruitful agenda to further explore elements that may shape the strategic and dynamic relationships in exhaustible-resource markets.

5 Appendix: Buyer's continuous-time consumption rule

The buyer's equilibrium payoff in continuous time satisfies

$$V(Q_t) = \int_t^T [U(q_\tau) - p_\tau q_\tau] e^{-\delta(\tau - t)} d\tau + e^{-\delta(T-t)} \frac{W(\bar{q})}{\delta} \tag{11}$$

where the choices at time points are evaluated along the equilibrium path (recall that $W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$). When time is discrete and the period length is Δ, the payoff to the buyer at stock level Q_t can be expressed as

$$V(Q_t) = [U(q_t) - p_t q_t]\Delta + e^{-\delta\Delta} V(Q_t - \Delta q_t). \tag{12}$$

For a small Δ, this equation can be Taylor approximated as

$$0 = [U(q_t) - p_t q_t]\Delta - \Delta\delta e^{-\delta\Delta} V(Q_t - \Delta q_t) - \Delta e^{-\delta\Delta} q_t V_Q(Q_t - \Delta q_t).$$

In the continuous-time limit, $\Delta \to 0$,

$$\delta V(Q_t) = [U(q_t) - p_t q_t] - q_t V_Q(Q_t).$$

The equilibrium choice of q_t maximizes the right hand side of (12), and satisfies

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t]\Delta - \Delta e^{-\delta\Delta} V_Q(Q_t - \Delta q_t) = 0, \tag{13}$$

or, in the limit $\Delta \to 0$,

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] - V_Q(Q_t) = 0. \tag{14}$$

To find an expression for $V_Q(Q_t)$, totally differentiate the equilibrium value function (11) to get

$$
\begin{aligned}
dQ_t V_Q(Q_t) &= dV = \\
& \int_t^T [U'(q_\tau) - p_\tau - \frac{\partial p_\tau}{\partial q_\tau} q_\tau] dq_\tau e^{-\delta(\tau - t)} d\tau + && (15) \\
& e^{-\delta(T-t)}[U(q_T) - p_T q_T - U(\bar{q}) + \bar{p}\bar{q}] dT - && (16) \\
& \int_t^T q_\tau dp_\tau e^{-\delta(\tau - t)} d\tau. && (17)
\end{aligned}
$$

By the fact that we are considering q_t along the equilibrium path, the expression on line (15) is zero: marginal perturbation of the choice variable yields a zero improvement in the value. In addition, since the buyer switches to the substitute at T, we have $q_T = \bar{q}$ and $p_T = \bar{p}$; hence, the value of the expression on line (16) is also zero. For the last term, note that the consumption cost can be written as

$$\int_t^T q_\tau p_\tau e^{-\delta(\tau - t)} d\tau = e^{-\delta(T-t)} \bar{p} \int_t^T q_\tau d\tau = e^{-\delta(T-t)} \bar{p} Q_t,$$

because equilibrium prices grow at the rate of interest. Therefore, the differential on line (17) captures the effect coming from postponement of the choke price. Thus, the expression for $dQ_t V_Q(Q_t)$ simplifies to

$$dQ_t V_Q(Q_t) = -\delta e^{-\delta(T-t)} \bar{p} Q_t dT = -\delta p_t Q_t d\tau, \tag{18}$$

where $dT = d\tau$, because marginal increase in T equals the period length $d\tau$. Since also $dt = d\tau$, and $dQ_t = -q_t dt$, we can write (18) as

$$V_Q(Q_t) = \delta p_t \frac{Q_t}{q_t}. \tag{19}$$

Reconsider now the first-order condition (13), and use $dt = \Delta$ to rewrite it as

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] dt - e^{-\delta dt} V_Q(Q_t - dt q_t) dt = 0.$$

Evaluate (19) at Q_{t+dt} and rewrite the first-order condition further to obtain

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] - e^{-\delta dt} \delta p_{t+dt} \frac{Q_{t+dt}}{q_{t+dt}} = 0.$$

As $dt \to 0$, the direct price effect is zero in equilibrium, $\partial p_t / \partial q_t = 0$. The effect on the price works through Q_{t+dt}, as it determines what the market can expect to receive in the future. Then, as $dt \to 0$, the equilibrium consumption rule then becomes

$$U'(q_t) - p_t - \delta p_t \frac{Q_t}{q_t} = 0,$$

which completes the proof.

References

[1] Ausubel, L., and R. Deneckere. 1989. "Reputation and Bargaining in Durable-Goods Monopoly", *Econometrica* 57, 511-32.

[2] Bagnoli, M., S. W. Salant, J. E. Swierzbinski. 1989. "Durable-Goods Monopoly with Discrete Demand", *Journal of Political Economy*, 97, 1459-1478

[3] Bulow, J. I. 1982. "Durable-Goods Monopolists", *Journal of Political Economy* 90, 314–32.

[4] Chakravorty, U., J. Roumasset and K. P. Tse. 1997. "Endogenous Substitution of Energy Resources and Global Warming", *Journal of Political Economy* 105, 1201-1233.

[5] Coase, R. H. 1972. "Durability and Monopoly", *Journal of Law and Economics*, 15, 143–49.

[6] Dasgupta, P. S., and G. M. Heal. 1974. "The Optimal Depletion of Exhaustible Resources", *Review of Economic Studies* 41, 3-38.

[7] Dasgupta, P. S., and G. M. Heal. 1979. "Economic Theory and Exhaustible Resources ", Cambridge: Cambridge Univ. Press.

[8] Gerlagh, R., and M. Liski. 2008. "Strategic Resource Dependence", FEEM Working Paper No. 72.2008.

[9] Gul, F., H. Sonnenschein, and R. Wilson (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture", *Journal of Economic Theory* 39, 155-190.

[10] Hotelling, H. 1931. "The Economics of Exhaustible Resources", *Journal of Political Economy* 39, 137–75.

[11] Hörner, J., and M. Kamien. 2004. " Coase and Hotelling: A Meeting of the Minds", *Journal of Political Economy* 112, 718-723.

[12] Kahn, C. M. 1986. "The Durable Goods Monopolist and Consistency with Increasing Costs", *Econometrica* 54, 275–94.

[13] Karp, L. 1996. "Depreciation Erodes the Coase Conjecture ", *European Economic Review* 40, 473-490.

[14] Karp, L., and D. Newbery. 1993. "Intertemporal Consistency Issues in Depletable Resources"˙In Handbook of Natural Resource and Energy Economics, vol. 3, edited by Allen V. Kneese and James L. Sweeney. Amsterdam: Elsevier Science.

[15] Levine, D. K., and W. Pesendorfer. 1995."When Are Agents Negligible ", *American Economic Review* 85, 1160-1170.

[16] Maskin, E., and D. Newbery. 1990." Disadvantageous Oil Tariffs and Dynamic Consistency ", *American Economic Review* 80, 143-156.

[17] McAfee, P., and T. Wiseman. 2008. "Capacity Choice Counters the Coase Conjecture", *Review of Economic Studies* 75, 317-332.

[18] Nordhaus, W., 1973. "The Allocation of Energy Reserves", Brookings Papers 3, 529-570.

[19] Sobel, J. 1991."Durable Goods Monopoly with Entry of New Consumers", *Econometrica* 59, 1455-1485.

[20] Stokey, N. L. 1981. "Rational Expectations and Durable Goods Pricing", *Bell Journal of Economics* 12, 112–28.